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Master＇s Thesis
석사 학위논문

# A geometric path tracking control of an autonomous vehicle for reduced sideslip effect 

Sung Hoon Youn（윤 성 훈 尹 星 量）

Department of
Information \＆Communication Engineering

## DGIST

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Advisor: Professor Kyoung-Dae Kim Co-advisor: Doctor Je Seok Kim<br>by<br>Sung Hoon Youn<br>Department of Information \& Communication Engineering<br>DGIST

A thesis submitted to the faculty of DGIST in partial fulfillment of the requirements for the degree of Master of Science in the Department of Information \& Communication Engineering. The study was conducted in accordance with Code of Research Ethics ${ }^{1}$
07. 02. 2021

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[^0]
# A geometric path tracking control of an autonomous vehicle for reduced sideslip effect 

Sung Hoon Youn

Accepted in partial fulfillment of the requirements for the degree of Master of Science.
05.25. 2021

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MS/IC 윤 성 훈. Sung Hoon Youn. A geometric path tracking control of an autonomous vehicle


#### Abstract

This paper introduces a technique to complement the tracking limitations of the pure pursuit controller for autonomous vehicles. We introduce two limitations of pure pursuit. The first limitation is the lateral sideslip effect of tires. Since pure pursuit is based on a geometry bicycle model, lateral sideslip effects of tires are reflected as a tracking error without any consideration. The second limitation is cutting corner effects. Pure pursuit describes the relation between the target point on the path and the rear wheel axis in terms of steering angle. Therefore, it does not consider the path's shape or the distance between the nearest path and the vehicle. To solve this problem and increase accuracy, the desired vehicle speed is adjusted by limiting the tire sideslip angle due to the curvature of the path. In addition, the tire sideslip effects are eliminated by adding a tire side slip compensator of a feedforward controller. The proposed technique was tested on a virtual DGIST campus track using Carmaker and Autoware. The result is faster and more accurate than existed works.


Keywords: An autonomous vehicle, Vehicle dynamics and control, Path tracking control, Pure pursuit, Sideslip effect, Lookahead distance

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## 1. INTRODUCTION

Autonomous driving will create new mobility concepts and opportunities and expand transportation system capacity and efficiency. These technologies will radically change the transportation infrastructure and influence plans. Autonomous vehicle adoption in future smart cities has to do with many potential benefits [1]. Autonomous driving technology consists of perception, motion planning, and control. Perception uses sensors to scan and monitor the environment [2]. Motion planning works to create a path or trajectory, and control precisely follows the path received from the motion planner. In this paper, we investigate a path tracking controller in control. The path tracking controller commands the vehicle's steering angle along the planned path selected toward zero lateral error and zero heading error between vehicle and selected path. These controllers can be classified by the applied vehicle model. The first is a geometric path tracking model that utilizes the geometric relationship between vehicle and path to create control laws for path tracking [3]. The geometry path tracking controllers work well in many autonomous driving situations because of easy implementation and low computational cost [20]. However, it may have poor performance around tight curved roads or at high speed because of a lack of considering dynamics [4]. The Second is a model based path tracking method. These methods consider the kinematic and dynamic vehicle model, which is possible to predict and compensate errors. The famous controller is Model predictive control (MPC) which has been widely used to forecast the future state of dynamics of the vehicle over a finite time range [5]. However, this control of formulations utilizes vehicle models to solve optimization problems in real time. Accordingly, it must be balancing model accuracy and computational efficiency [6]. Another problem is that the optimization can fail if the initial values are inappropriate and the computation time for each step is unpredictable. As a result, most MPC controllers have only been verified through simulation [4]. The geometry path tracking controller is generally high preferred than the model-based path tracking controller in urban driving environments [7]. Because firstly, the driving speed is low, the geometry path tracking controller works well at a low speed. Secondly, localization error can be high, and the path is non-smooth waypoints, So a robust path tracking controller is needed. Thirdly, computation time is very low rather than model-based path tracking controller, which means we can use computing power for other autonomous driving functions like perception. Therefore, in this paper, we use the geometry path tracking controller. However, the speed limit in the urban area generally is $50 \mathrm{~km} / \mathrm{h}$ [21]. Accordingly, When a vehicle drives on a high curved path with $50 \mathrm{~km} / \mathrm{h}$, the geometry path tracking controller can not ensure exact accuracy because of a lack of consideration of vehicle dynamics. Therefore, we propose a geometry path tracking controller considering sideslip effects. Which also can be applied not only urban area but also high-speed area like a highway. There are pure pursuit and Stanley method [22] in geometry path
tracking controllers. Pure pursuit has better tracking performance than the Stanley method when the vehicle speed is high [23]. Also, the Stanley method is not compatible with discrete paths, but the pure pursuit is robust in discrete paths[3]. Therefore we choose the pure pursuit path tracking controller. The pure pursuit controller assumes that the vehicle follows the Ackerman geometry. It calculates the necessary steering angle of the front wheel when the lookahead point and the rear wheel are on the same virtual circle by selecting the lookahead point on the path as a design parameter. Pure pursuit controller has changeable performance characteristics depends on lookahead distance. This characteristic is a trade-off between accuracy and stability [3]. Research has been conducted to improve path tracking performance using this characteristic. [12] define the cross track error between the path and the vehicle and optimizes the lookahead distance using fuzzy logic. [13] shows the lookahead distance as a 2 nd order polynomial function in consideration of the relationship between vehicle characteristics and path. The most negative effect caused by the characteristics of the lookahead distance is the cutting corner effect on the path. As a study to improve this [14, 15] defined the distance of the waypoint on the path closest to the vehicle as an error and improved it using a PID controller. However, there is one more limitation. Since the pure pursuit controller is a geometry based model, tire slip angle is not considered. That leads to a tracking error, and the influence increases as the vehicle speed and curvature increase. Therefore, this paper determines the maximum vehicle speed by limiting the tire slip angle according to the curvature. In addition, to compensate for the side slip effect, a feed forward controller is combined. The proposed method has three advantages. The first advantage is to reduce the influence of sideslip effects that are not considered in pure pursuit. The second advantage, reducing the aspect of cutting corners because the lookahead distance decreases as speed is reduced on the curvature paths. The third advantage is that it takes significant effort to tune the heuristics to get the satisfactory performance of the path tracking controller [16]. This technique has an advantage in implementation because there are no heuristic elements except for the lookahead distance threshold. The proposed technique was tested on a virtual DGIST campus track in the Carmaker environment. Autoware was used as an autonomous driving system software to verify the proposed technique. It was proved that when the proposed method was added, the path tracking performance was significantly improved, and the driving speed increased compared to the basic pure pursuit or the existed work.

## II. Vehicle Models

This section introduces two vehicle models. First is the kinematic planar bicycle model used to correct the tire sideslip angle. The second is the dynamic planar bicycle model for linear analysis of pure pursuit path tracking controllers.

### 2.1. Kinematic planar bicycle model with sideslip

The main assumption of a kinematic planar bicycle model should be equal to the tire's speed vector. However, it is suitable for slow motion. The effects of tire sideslip cannot be ignored at high speeds. Accordingly, modeling is performed by adding tire sideslip angles. However, the dynamic movement of the vehicle is better captured by the model based on force constraints rather than kinematic constraints [17]. However, the pure pursuit is designed based on Ackerman geometry. Therefore, the method of adding a slip angle term in a kinematic bicycle model can be easily applied to the pure pursuit controller law. Therefore, this method is a straightforward approach to compensate for the side slip effect that occurs as the speed increases. A schematic model is shown in Figure $1 . \delta$ is a steering angle, $l_{f}$ is the distance of front axle to c.g., $l_{r}$ is the distance of rear axle to c.g., $\alpha_{f}$ is front sideslip angle that is between front tire orientation and velocity vector, $\alpha_{r}$ is rear sideslip angle that is rear tire orientation and velocity vector, $\beta$ is vehicle sideslip angle, $\psi$ is yaw angle. According to the Ackerman rule, an instantaneous rolling $O$ of the vehicle exists in the direction of adding the tire slip angle from the vertical component of each tire to the vehicle. $R$ is the radius of rear axle to $O$. that is perpendicular at rear tire velocity vector, and $R^{\prime}$ is the radius between $O$.


Figure 1 Schematic of kinematic planar bicycle model with sideslip

Our purpose is a relation with the steering angle $\delta$ and the radius $R$. This can be obtained through the sine rule. The derived equation is

$$
\begin{equation*}
\cos \left(\alpha_{r}\right)\left\{\tan \left(\alpha_{f}+\delta\right)-\tan \left(\alpha_{r}\right)\right\}=\frac{l_{f}+l_{r}}{R} \tag{1}
\end{equation*}
$$

This paper considers tires sideslip angle is small because of the cornering stiffness, which covered next section, is constant. that means a small angle assumption is possible.

$$
\begin{equation*}
\delta=\tan ^{-1}\left(\frac{l_{f}+l_{r}}{R}+\alpha_{r}\right)-\alpha_{f} \tag{2}
\end{equation*}
$$

### 2.2. Dynamic planar bicycle model in terms of errors on straight path

For the linear analysis of the pure pursuit controller, a dynamic planar bicycle model with between vehicle and desired path as error states are derived. Considering the force in the body fixed dynamic model as shown in Figure 2. $F_{x f}$ is the longitudinal forces created by the front tires, $F_{y f}, F_{y r}$ are lateral forces created by the front, rear tires, and $\dot{\psi}$ is yaw rate. The equation of motion below was derived using the Newton-Euler method and expressed as a constant longitudinal velocity $v_{x}$ [9].


Figure 2 Schematic of planar bicycle model

$$
\begin{gather*}
m \dot{v}_{y}=F_{y r}+F_{x f} \sin (\delta)+F_{y f} \cos (\delta)-m \dot{\Psi} v_{x}  \tag{3}\\
I_{z} \ddot{\Psi}=l_{f} F_{x f} \sin (\delta)+l_{f} F_{y f} \cos (\delta)-l_{y} F_{y r} \tag{4}
\end{gather*}
$$

Tire slip angle is the angle between the tire's speed direction and the tire's longitudinal axis. Therefore, the slip angle of each tire can be expressed as

$$
\begin{gather*}
\alpha_{f}=\tan ^{-1}\left(\frac{v_{y}+\dot{\Psi} l_{f}}{v_{x}}\right)-\delta  \tag{5}\\
\alpha_{r}=\tan ^{-1}\left(\frac{v_{y}-\dot{\Psi} l_{r}}{v_{x}}\right) \tag{6}
\end{gather*}
$$

The lateral force exerted on the tire depends on the degree of lateral deflection on a thread in the contact path [8]. The lateral force is proportional to the slip angle.

$$
\begin{align*}
& F_{y f}=-C_{f} \alpha_{f}  \tag{7}\\
& F_{y r}=-C_{r} \alpha_{r} \tag{8}
\end{align*}
$$

Substituting (5), (6), (7), (8) into equation(3), (4). Assuming that $\delta, \alpha_{f}, \alpha_{r}$ are small and $F_{x f}$ is 0 , we can obtain

$$
\begin{gather*}
m \dot{v}_{y}=-C_{r}\left(\frac{v_{y}-\dot{\psi} l_{r}}{v_{x}}\right)-C_{f}\left(\frac{v_{y}+\dot{\Psi} l_{f}}{v_{x}}\right)-m \dot{\Psi} v_{x}+C_{f} \delta  \tag{9}\\
I_{z} \ddot{\Psi}=-l_{f} C_{f}\left(\frac{v_{y}+\dot{\psi} l_{f}}{v_{x}}\right)+l_{r} C_{r}\left(\frac{v_{y}-\dot{\Psi} l_{r}}{v_{x}}\right)+l_{f} C_{f} \delta \tag{10}
\end{gather*}
$$

In [8], it can be expressed by terms of errors on a straight path. $e^{\prime}$ is the distance of the $c . g$. of the vehicle from the path, $\theta_{e}$ is the orientation error of the vehicle with respect to the path. it can be considered as.

$$
\begin{align*}
& \ddot{e^{\prime}}=v_{y}+v_{x} \dot{\theta}_{e}, \dot{\theta_{e}}=\dot{\Psi}  \tag{11}\\
& \dot{e}^{\prime}=v_{y}+v_{x} \theta_{e}, \quad \theta_{e}=\Psi \tag{12}
\end{align*}
$$

Substituting (11) and (12) into equations (9) and (10), the following state space equation can be obtained.

$$
\frac{d}{d t}\left[\begin{array}{c}
e^{\prime}  \tag{13}\\
\dot{e}^{\prime} \\
\theta_{e} \\
\dot{\theta}_{e}
\end{array}\right]=\left[\begin{array}{cccc}
0 & 1 & 0 & 0 \\
0 & -\frac{C_{f}+C_{r}}{m v_{x}} & \frac{C_{f}+C_{r}}{m} & \frac{-C_{f} l_{f}+C_{r} l_{r}}{m v_{x}} \\
0 & 0 & 0 & 1 \\
0 & -\frac{C_{f} l_{f}-C_{r} l_{r}}{I_{z} v_{x}} & \frac{C_{f} l_{f}-C_{r} l_{r}}{I_{z}} & -\frac{C_{f} l_{f}^{2}+C_{r} l_{r}^{2}}{I_{z} v_{x}}
\end{array}\right]\left[\begin{array}{c}
e^{\prime} \\
\dot{e}^{\prime} \\
\theta_{e} \\
\dot{\theta}_{e}
\end{array}\right]+\left[\begin{array}{c}
0 \\
\frac{c_{f}}{m} \\
0 \\
\frac{c_{f} l_{f}}{m}
\end{array}\right] \delta
$$

## III. Analysis pure pursuit

In this section, the controller law of a pure pursuit is reconstructed into terms related to errors. Using the dynamic model of section 2.2, we derive the optimal lookahead distance for each vehicle speed through eigenvalues plot. Also, based on the analysis, the limitations of pure pursuit are explained. $\theta$ is the angle between the goal point and the vehicle's heading.

### 3.1. Pure pursuit control law

The pure pursuit controller is based on the vehicle's rear axle and can select the path's goal point by a look ahead distance. The goal point and the point of the vehicle's rear axle can be on an arbitrary circle. The radius of this circle is used to determine the vehicle's steering angle. Pure pursuit control law is as follows [3].

$$
\begin{gather*}
R=\frac{l_{d}}{2 \sin (\theta)}  \tag{14}\\
\delta=\tan ^{-1}\left(\frac{2 L \sin (\theta)}{l_{d}}\right) \tag{15}
\end{gather*}
$$

### 3.2. Pure pursuit decomposition with errors

The effect of lateral error and heading error is analyzed as follows. In Figure 3, the path is assumed to be a complete circle. $e$ is defined as the closest distance on the path based on the vehicle's rear wheel axis. Moreover, $\theta_{e}$ is the angle difference between the vehicle's heading direction and the tangent line of the path. According to (15), the control law is as follows.

$$
\begin{equation*}
\delta=\tan ^{-1}\left(\frac{2 L \sin \left(\theta+\theta_{e}\right)}{l_{d}}\right) \tag{16}
\end{equation*}
$$

$\theta$ can be expressed as

$$
\begin{equation*}
\theta=\sin ^{-1}\left(\frac{e_{g}}{l_{d}}\right) \tag{17}
\end{equation*}
$$



Figure 3 Schematic of pure pursuit with errors on a curved path
$e_{g}$ is derived as follows.

$$
\begin{gather*}
x_{1}=R+e-e_{g}  \tag{18}\\
x_{1}^{2}+y_{1}^{2}=R^{2}  \tag{19}\\
l_{d}^{2}=e_{g}^{2}+y_{1}^{2} \tag{20}
\end{gather*}
$$

If $x_{1}$ and $y_{1}$ are eliminated using (18), (19), (20), the following equation can be obtained.

$$
\begin{equation*}
e_{g}=\frac{l d^{2}+2 R e+e^{2}}{2(R+e)} \tag{21}
\end{equation*}
$$

Substituting (17) and (21) into (16) can be expressed as follows. In this case, it is assumed that A and $\theta_{e}$ are small angles.

$$
\delta=\frac{2 L}{l_{d}}\left(\frac{l_{d}^{2}+2 R e+e^{2}}{2 l_{d}(R+e)}+\theta_{e}\right)
$$

Rearranged Form is as follows.

$$
\begin{equation*}
\delta=L\left(\frac{1}{R+e}+\frac{e}{l_{d}^{2}}\left(1+\frac{R}{R+e}\right)+\frac{2 \theta_{e}}{l_{d}}\right) \tag{22}
\end{equation*}
$$

Compared to $R$, the value of e is very small. Therefore, assume that $R+e \cong R$

$$
\begin{equation*}
\delta=L\left(\kappa+\frac{2 e}{l_{d}^{2}}+\frac{2 \theta_{e}}{l_{d}}\right) \tag{23}
\end{equation*}
$$

Considering (23), pure pursuit can be independently divided into components of path curvature $\kappa$, lateral error $e$, and heading error $\theta_{e}$. As lookahead distance increases, the lateral error decreases in proportion to the square, and the heading error is decreased proportionally. We can use this equation to perform linear analysis.

### 3.3. Pure pursuit analysis on straight path

This section analyzes how performance varies according to the lookahead distance through eigenvalues plot. From this, an optimal lookahead distance is obtained according to the speed. before applying (23) to (13), we need to set the error measurement position. Since the lateral error has to be calculated from $c . g$. the relational equation is $e=e^{\prime}+l_{r} \theta_{e}$. Also, $\kappa=0$ because of considering a straight path. then the equation (23) can be a below equation

$$
\begin{equation*}
\delta=2 \mathrm{~L}\left(\frac{\mathrm{e}^{\prime}-\mathrm{l}_{\mathbf{r}} \boldsymbol{\theta}_{\mathrm{e}}}{\mathrm{l}_{\mathrm{d}}^{2}}+\frac{\boldsymbol{\theta}_{\mathbf{e}}}{\mathrm{l}_{\mathrm{d}}}\right) \tag{24}
\end{equation*}
$$

Substituting (24) into (13) can be expressed as follows.

$$
\left[\begin{array}{cccc}
0 & 1 & 0  \tag{25}\\
-\frac{2 \mathrm{LC}_{\mathrm{f}}}{\mathrm{ml}_{\mathrm{d}}^{2}} & -\frac{\mathrm{C}_{\mathrm{f}}+\mathrm{C}_{\mathrm{r}}}{m v_{\mathrm{x}}} & \frac{\mathrm{C}_{\mathrm{f}}+\mathrm{C}_{\mathrm{r}}}{m}-\frac{2 \mathrm{LC}_{\mathrm{f}}}{m}\left(\frac{1}{\mathrm{l}_{\mathrm{d}}}-\frac{\mathrm{l}_{\mathrm{r}}}{\mathrm{l}_{\mathrm{d}}^{2}}\right) & \frac{-\mathrm{C}_{\mathrm{f}} \mathrm{l}_{\mathrm{f}}+\mathrm{C}_{\mathrm{r}} \mathrm{l}_{\mathrm{r}}}{\mathrm{mv}_{\mathrm{x}}} \\
0 & 0 & 0 & 1 \\
-\frac{2 \mathrm{LC}_{\mathrm{f}} \mathrm{l}_{\mathrm{f}}}{\mathrm{I}_{\mathrm{z}} \mathrm{l}_{\mathrm{d}}^{2}} & -\frac{\mathrm{C}_{\mathrm{f}} \mathrm{l}_{\mathrm{f}}-\mathrm{C}_{\mathrm{r}} \mathrm{l}_{\mathrm{r}}}{\mathrm{I}_{\mathrm{z}} \mathrm{v}_{\mathrm{x}}} & \frac{\mathrm{C}_{\mathrm{f}} \mathrm{l}_{\mathrm{f}}-\mathrm{C}_{\mathrm{r}} \mathrm{l}_{\mathrm{r}}}{\mathrm{I}_{\mathrm{z}}}-\frac{2 \mathrm{LC}_{\mathrm{f}} \mathrm{l}_{\mathrm{f}}}{\mathrm{I}_{\mathrm{z}}}\left(\frac{1}{\mathrm{l}_{\mathrm{d}}}-\frac{\mathrm{l}_{\mathrm{r}}}{\mathrm{l}_{\mathrm{d}}^{2}}\right) & -\frac{\mathrm{C}_{\mathrm{f}} \mathrm{l}_{\mathrm{f}}^{2}+\mathrm{C}_{\mathrm{r}} \mathrm{l}_{\mathrm{r}}^{2}}{\mathrm{I}_{\mathrm{z}} \mathrm{v}_{\mathrm{x}}}
\end{array}\right]
$$

Eigenvalues are obtained from (25) and vehicle parameters in Table 1. Figure 4 shows eigenvalues of the vehicle speed from $1 \mathrm{~m} / \mathrm{s}$ to $10 \mathrm{~m} / \mathrm{s}$ when the lookahead distance is 1 m . The settling time is different depending on the vehicle's speed. Moreover, if the vehicle is above a certain speed, it will be unstable. Therefore, the lookahead distance is obtained according to the function of speed when the settling time is the smallest. It is used to determine the lookahead distance according to the speed during driving. However, some unstable factors are not considered in the modeling, such as path waypoints interval and time delay for steering. In addition, when the lookahead distance is large, the cutting corner effect, which will be explained in the following section, occurs. Therefore, we influence these factors is reduced by offsetting and thresholding the obtained lookahead distance function. A tuned lookahead function is shown in figure 5 and is expressed by the following equation.

$$
\begin{gather*}
l_{d}=0.00025 v_{x}^{3}+0.0427 v_{x}^{2}+0.0798 v_{x} \\
l_{\text {dtuned }}= \begin{cases}l_{d}+1 & l_{d} \leq 11 \\
12 & l_{d}>11\end{cases} \tag{26}
\end{gather*}
$$



Figure 4 Vehicle eigenvalues as speed increased


Figure 5 Lookahead distance function of speed

### 3.4. Two pure pursuit limitations

The pure pursuit has two limitations. The first is the tire's sideslip effect. As can be seen from Figure 3, tire sideslip angle is not considered in the control law. Therefore, as the speed and curvature increase, the tracking error due to the sideslip effect increases. The second is a phenomenon known as the cutting corner effect. The previous section shows that if the curvature is constant, errors can be defined, and it can be converged to zero. However, if the curvature is variable, such as the path is clothoid, lateral error $e$ in equation (23) is not defined. Therefore, that causes tracking errors. The best way is to reduce the cutting corner effect is to perform the lookahead distance as short as possible because the short lookahead distance makes the curvature of the path closest to the vehicle and the path curvature of the goal point almost the same. But, short lookahead distance means low vehicle speed, as shown in Figure 4. We can consider unmodeled components that are not considered in the pure pursuit model as disturbances referring to Figure 6.


Figure 6 Pure pursuit control architecture with disturbance

Therefore, in this paper, our proposed method reduces these two limitation factors. The sideslip effect is reduced by limiting vehicle speed and compensating sideslip effect. The cutting corner effect is reduced by speed limiting on the curvature path.

## IV. The Proposed Method

In this section, our proposed method will be introduced. Our proposed method can reduce two unmodeled factors mentioned above. This method restricts maximum speed by limiting sideslip angle on tires. To do this, sideslip angles are set as a design parameter. Then, maximum curvature is searched on the path within pre-defined distance. The obtained maximum curvature is used to determine the vehicle's limited speed that has two advantages. Firstly, the vehicle can drive within the sideslip angles that are defined as the design parameter. Secondly, the vehicle speed decreases according to maximum curvature, the lookahead distance decreases, and the cutting corner effect also decreases.

### 4.1 Controller architecture

For overall control architecture, as shown in Figure 7, driving path, current vehicle position and orientation, and vehicle speed information are required as input information. These are used by the speed limiter and pure pursuit tracking controller. The speed limiter uses the desired tire slip angle as a design parameter and determines the speed not to exceed the slip angles. The pure pursuit tracking controller sets the goal point according to the lookahead distance and determines the steering angle for driving from the vehicle's current position to the goal point. The sideslip compensator estimates a tire slip angle and reduced side slip effect.


Figure 7 Overall control architecture.

### 4.2. Speed limiter

The speed limiter operates to restrict the slip angle that can be applied to tires. It needs a maximum curvature on a given path. The maximum curvature is obtained within a search distance. That is calculated using a distance when the vehicle is stopped from current velocity. An equation is shown at (27). $v_{c}$ is current vehicle velocity and $a_{\max }$ is maximum longitudinal deceleration of the vehicle.

$$
\begin{equation*}
s_{d}=\frac{V_{c}^{2}}{2 a_{\max }} \tag{27}
\end{equation*}
$$

We consider steady state cornering for the speed limiter design. $\dot{v}_{y}=0, \ddot{\Psi}=0, \dot{\psi}=\kappa v_{x}, F_{x f}=0$ are considered, and Equations (3) and (4) are expressed as follows.

$$
\begin{align*}
& m v_{x}^{2} \kappa=F_{y r}+F_{y f}  \tag{28}\\
& 0=l_{f} F_{y f}-l_{y} F_{y r} \tag{29}
\end{align*}
$$

Following equations can be derived using (7), (8), (28), (29).

$$
\begin{gather*}
\alpha_{f}=-\frac{l_{f} m v_{x}^{2} \kappa}{C_{f} L}  \tag{30}\\
\alpha_{r}=-\frac{l_{r} m v_{x}^{2} \kappa}{C_{r} L} \tag{31}
\end{gather*}
$$

If we consider the vehicle is understeer setting that is $\alpha_{f}>\alpha_{r}$, the speed limiter is constructed using front side slip angle (32).

$$
\begin{equation*}
v_{\text {limit }}=\sqrt{\frac{\alpha_{d} C_{f} L}{l_{f} m}} \frac{1}{\sqrt{\left|\kappa_{\max }\right|}} \tag{32}
\end{equation*}
$$

$v_{\text {limit }}$ means limited speed of vehicle. $\kappa_{\max }$ is the maximum curvature that can be obtained through path within $s_{d} . \alpha_{d}$ is the desired maximum sideslip angle.

## 4.3. lateral tire slip compensator

In steady-state condition, considering sideslip angle, the rear wheel velocity vector is not anymore vehicle heading. If we consider the rear wheel velocity vector considering the sideslip angle, (14) represents the below equation.

$$
\begin{equation*}
\mathrm{R}=\frac{l_{d}}{2 \sin \left(\theta-\alpha_{r}\right)} \tag{33}
\end{equation*}
$$

Substituting (33) into (2), we can derive a steering control equation that compensates for the sideslip angle.

$$
\begin{equation*}
\delta=\tan ^{-1}\left(\frac{2 L \sin \left(\theta-\alpha_{r}\right)}{l_{d}}+\alpha_{r}\right)-\alpha_{f} \tag{34}
\end{equation*}
$$

$\alpha_{f}$, and $\alpha_{r}$ can be calculated through (30) and (31), $v_{x}$ is obtained using (32) and $\kappa$ is the curvature of path. According to [11], when the velocity vector is feedback as the velocity increases, it tends to become unstable as the velocity increases. Therefore, to prevent it, use $v_{d}$ that is desired vehicle velocity instead of current vehicle velocity and The overall steering control equation is below.

$$
\begin{equation*}
\delta=\tan ^{-1}\left(\frac{2 L \sin \left(\theta+\frac{l_{r} m v_{d}^{2} \kappa}{C_{r} L}\right)}{l_{d}}-\frac{l_{r} m v_{d}^{2} \kappa}{C_{r} L}\right)+\frac{l_{f} m v_{d}^{2} \kappa}{C_{f} L} \tag{35}
\end{equation*}
$$

## 5. Simulation

### 5.1 Experimental setup

The experiment is conducted in an autonomous driving test simulation environment that integrates Carmaker and Autoware[19]. Path tracking controller receives and processes vehicle location and speed information at 50 Hz intervals and transmits steering angle and vehicle speed data at 50 Hz intervals. Information on the vehicle's characteristics is shown in Table 1. Cornering stiffness is obtained from [10] in Table 1. Also, cornering stiffness is valid when the sideslip angle is within $\pm 2$ degrees. As shown in Figure 8 , it is conducted in the DGIST campus map environment, and the test is conducted by following the centerline of the lane. There are four areas on this map that are useful for testing. The first section consists of a straight road after cornering on a straight road to check the cutting corner and side slip effects. Section 1 consists of between 400 m and 600 m on the track. The second section is a circular path consisting of $\mathrm{R}=135 \mathrm{~m}, \mathrm{R}=240 \mathrm{~m}$, and $\mathrm{R}=260 \mathrm{~m}$. The cutting corner problem is not observed in the circular path. Therefore, only the sideslip effects can be seen. We will discuss this topic in following test. Section 2 consists of between 770 m and 1140 m on the track. The third section consists of with variable curvature path, therefore, the cutting corner phenomenon and side slip will be observed. Section 3 consists of between 1140 m and 1210 m on the track. The fourth section is the path with the largest curvature in the track. Driving the vehicle with pure pursuit has significant cutting corner and side slip effects when entering at high speed. Section 4 consists of between 1370 m and 1425 m on the track.


Figure 8 DGIST campus map

| Symbol | Description | Value |
| :--- | :--- | :--- |
| $m$ | Vehicle mass | $1319.9[\mathrm{~kg}]$ |
| $l_{f}$ | Distance of front axle from c.g | $1.33[\mathrm{~m}]$ |
| $l_{r}$ | Distance of rear axle from c.g. | $1.37[\mathrm{~m}]$ |
| $C_{f}$ | Estimated Front Cornering stiffness | $69783[\mathrm{~N} / \mathrm{rad}]$ |
| $\mathrm{C}_{\mathrm{r}}$ | Estimated Rear Cornering stiffness | $74744[\mathrm{~N} / \mathrm{rad}]$ |
| $I_{z}$ | Moment of Inertia | $2600\left[\mathrm{kgm}{ }^{2}\right]$ |

Table 1 Vehicle Parameters

### 5.2 Simulation results

### 5.2.1 Functional Test

Functional Test verifies the functional aspect of the proposed method. We checked the performance when the speed limiter and side slip compensator were introduced. The first controller to be compared in test 1 is the Autoware Pure Pursuit (PP) [18]. This method uses a basic pure pursuit equation (15) for steering command. In this test, lookahead distance is defined as proportional to the vehicle speed. The second controller is the proposed method without the sideslip compensator(PMW). It is tested to check the effect of the sideslip compensator. The third controller is the entire proposed method(PM). In PP test, the vehicle drives at $1 \mathrm{~m} / \mathrm{s}$, which means there is no sideslip effect. The purpose of PP test is to check the cutting corner effect. Therefore the lookahead ratio is set to 12 times the vehicle drive velocity. In PMW and PM test, the maximum vehicle velocity is $22.22 \mathrm{~m} / \mathrm{s}$, and $\alpha_{d}$ is 1 degree and consider $v_{d}$ as $v_{\text {limit }}$. The result is shown in Figure 9 . The test 1 is conducted on the track between 400 m and 1425 m . In the second section on the track ( $700 \sim 1140 \mathrm{~m}$ ), It is observed that no lateral and heading error except changed path radius in PP. that means there is no cutting corner effect on the constant curvature path that is long the vehicle states can be considered in steady state condition. However, path tracking errors are observed in PMW. We can notice from the result of PP that these errors are affected by the lateral side slip effect. Path tracking errors are reduced in PM as compensating side slip effects. In Figure 9, $\mathrm{PM}^{-}$indicates estimated and compensated side slip angle. In the first section of the track ( $400 \mathrm{~m} \sim 600 \mathrm{~m}$ ) and the Fourth section of the $\operatorname{track}(1370 \mathrm{~m} \sim 1425 \mathrm{~m})$ consist of variable and high curvature paths. Significant errors are observed in PP because the lookahead distance is 10 m long compared to paths. However, PM and PMW have reduced errors compared with PP. That means the speed limiter works to restrict sideslip angle within 1 degree. As a result, vehicle speed and lookahead distance are reduced. Moreover, the cutting corner effect is reduced.

| Functional Test |  |  |
| :---: | :---: | :---: |
| Method | Design Parameter |  |
| Pure pursuit [20] (PP) | $\mathrm{v}_{\mathrm{x}}$ | $1[\mathrm{~m} / \mathrm{s}]$ |
|  | $\mathrm{l}_{\mathrm{d}}$ | $12 \mathrm{v}_{\mathrm{x}}$ |
| Proposed method without slip com- |  |  |
| pensation (PMW) | $\alpha$ | $1[\mathrm{deg}]$ |
| Proposed method (PM) | $\mathrm{v}_{\max }$ | $22.22[\mathrm{~m} / \mathrm{s}]$ |
|  | $\alpha$ | $1[\mathrm{deg}]$ |
|  | $\mathrm{v}_{\max }$ | $22.22[\mathrm{~m} / \mathrm{s}]$ |

Table 2 Path tracking methods and parameters for functional test


Figure 9 Functional test experiment results

### 5.2.2 Performance Test

This test compares the proposed method's performance with other existing works [13][15]. The first controller is a pure pursuit with PID (PID). PID has the velocity planner to prevent slipping and rollovers, and a PID controller minimizes the lateral error to reduce the cutting corner effect. Since PID gain is not presented in [15], the gain was heuristically tuned through many experiments. The second controller is a pure pursuit with the proposed method (PM) in this paper. The desired maximum slip angle is 2 degrees, considering as, and the maximum vehicle velocity is $70 \mathrm{~km} / \mathrm{h}$, which is the same as PP. The third controller is the improved pure pursuit algorithm (IPP)[13]. This controller improved path tracking accuracy by adding vehicle characteristics and lateral error terms to look ahead distance. Design parameters were selected in the characteristics of the vehicle. These parameters are shown in table2. However, this paper does not introduce a speed profile. Since path tracking accuracy is significantly affected by vehicle speed, We add a simple speed profile from [24]. The lateral acceleration limit is seen in table 2. In this test, PID, PM, and IPP are compared with tracking performance and vehicle speed. The test is conducted on the DGIST campus track between 400 m and 1425 m . The result is shown in Figure 10. In the first section of the track $(400 \mathrm{~m} \sim 600 \mathrm{~m})$ and the Fourth section of the track ( $1370 \mathrm{~m} \sim 1425 \mathrm{~m}$ ), PM is better tracking performance than PID and IPP because PM's vehicle speed and lookahead distance are lower than PID. In the second section of the track ( $700 \mathrm{~m} \sim 1140 \mathrm{~m}$ ), PM is better path tracking performance than other controllers though vehicle speed is higher than other controllers because of the sideslip compensator. PM and PID have almost the same error size in the third section of the track ( $1140 \mathrm{~m} \sim$ $1210 \mathrm{~m})$. However, we can see IPP has a lower lateral error than PM. Because vehicle speed of PM is much faster than IPP and sideslip compensator does not work well in abrupt curvature change. In this test, PM is faster than PID. From 400 m to 1425 m , the overall driving time of PID is 67.2 seconds, PM is 64.6 seconds, and IPP is 85.25 seconds. Therefore, the PM is better performance than PID and IPP in path tracking performance and vehicle speed.

| Performance Test |  |  |
| :---: | :---: | :---: |
| Method | Design Parameter |  |
| Pure pursuit with PID [15] (PID) | PID gain | $\mathrm{P}=0.01$ |
|  |  | $\mathrm{I}=0.001$ |
| Proposed method (PM) | Desired sideslip limit angle | 2 [deg] |
| $2^{\text {nd }}$ polynomial lookahead func- <br> tion[13] with speed profile [24 ] <br> (IPP) | $2^{\text {nd }}$ polynomial lookahead <br> function | $\mathrm{A}=\frac{1}{6}, \mathrm{~B}=0.5$ |

Table 2 Path tracking methods and parameters for performance test


Figure 10 Performance test experiment results

## 6. CONCLUSION

This paper covers two pure pursuit limitations that are the sideslip effect and the cutting corner effect. To solve this, we introduced the speed limiter, which improves the path tracking performance by changing vehicle speed according to the curvature. In addition, a feedforward sideslip compensator was introduced to reduce the effects of the sideslip. As a result, the path tracking accuracy increased dramatically in the path where the path is not short and curvature is constant. Compared with the proposed method and the existed work, the proposed method can drive at a higher speed and accurate path tracking. However, an error occurs where the curvature is low and variable because the vehicle speed is set high in the low curvature. Therefore, future work is improving this problem.

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## 요 약 문

## 자율주행차량의 측면 슬립 효과를 줄이기 위한 기하학적 경로 추적 제어

이 논문에서는 자율주행차용 순수 추적 경로 추종 제어기의 한계를 보완하는 기술을 소개한다. 순수 추적 경로 추종 제어기는 두가지 성능 상 한계를 가진다. 첫 번째는 타이어 사이드 슬립 효 과를 고려하지 못하는 것이다. 순수 추적 경로 추적 제어기는 기하학 자전거 모델을 기반으로 하 기 때문에 타이어 측면 슬립효과는 고려하지 않아 경로 추종을 저해하는 역할을 한다. 두 번째는 커팅 코너 효과이다. 순수 추적 경로 추종 제어기는 경로의 목표 지점과 뒷바퀴의 방향의 사이각 을 제어에 활용한다. 따라서 목표지점 까지의 경로를 원으로 가정하기 때문에, 경로의 실제 모양 이나 가장 가까운 경로와 차량 사이의 거리는 고려하지 않는다. 이러한 문제를 해결하고 정확도 를 높이기 위해 경로의 곡률에 따라 타이어의 사이드 슬립 각도를 제한한다. 이는 곡률이 커짐에 따라 최대 속도를 줄이게 되면 목표거리도 줄어들어 커팅 코너 효과 또한 줄어든다. 또한 피드 포워드 컨트롤러의 타이어 사이드 슬립 각도 보상기를 구성하여 타이어 사이드 슬립 효과를 제거 한다. 제안된 기법은 카메이커와 오토웨어를 사용하여 가상 캠퍼스 트랙에서 테스트하였다. 그 결 과 기존 기술에 비해 주행 속도 및 경로 추종 정확도가 향상되었다.

핵심어 : 자율주행, 차량 동역학 및 제어, 경로 추종, 타이어 사이드 슬립, 목표점


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