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Modeling and Precision Stop Control of Metropolitan Train with Robust Output Feedback Model Predictive Control

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абстракт

This thesis apply robust output feedback model predictive control (ROFMPc) for precision stop control of metropolitan train. For modeling the train, we consider actuation delay of train’s actuator, switching of actuators, nonlinearity of actuator’s saturation, so we consider a train as an nonlinear switching system. However, we consider a train as a uncertain linear system, because it is possible to obtain the exact train’s actuator model. Therefore, ROFMPc is selected as controller for precision stop control of a train. We simulate precision stop control of a train with ROFMPc and evaluate the performance of ROFMPc. We compare ROFMPc and output feedback MPC (OFMPC) in performance of precision stop control of train. Also, we test for performance of ROFMPc under uncertain train’s mass and running resistance. Results show that ROFMPc is better than OFMPC and ROFMPc has robustness of uncertain train’s mass and running resistance. Because the state-feedback control’s structure is included in robust MPC (RMPC)’s structure, we compare state-feedback control and RMPC in performance of simple model with disturbance. Also, we simulate the precision stop control of train using state-feedback control with state estimation because the result of simulation of comparison of RMPC and state-feedback control shows state-feedback control’s step response is similar with RMPC’s step response for the simple model with disturbance. As a result, state-feedback control with state estimation is appropriate for precision stop control of train and state-feedback control with state estimation is robust in performance of precision stop control of train under uncertain train’s mass and running resistance.

Ключевые слова: robust output feedback model predictive control, metropolitan train, precision stop control, nonlinear switching system
부모님의 헌신과 사랑에 이 논문을 바칩니다.
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1

Introduction

1.1 Motivation

Many people use the metropolitan train for transportation, because the metropolitan train sometimes is faster than using the bus and very convenient for punctual arrival. According to statistics of Ministry of Land Infrastructure and Transport (MOLIT), about 2.5 billion people used the metropolitan train in 2015. Safety and timely function of metro train is therefore a significant matter.

The Korean government seeks to introduce a new type of train for reducing delay time of train in platform. Among many improvements, one of the most significant differences is its widened door. The widened door would allow three lines of passengers to move through simultaneously although the current door is designed for two lines of passengers to move through simultaneously. Therefore, if all metro train’s doors are changed widened door, delay time of the metro train in the platform would reduce.

Recently platform screen door (PSD), which divides passenger space and train track for safety, is created at most platforms in Korea. Therefore, metro train has to stop precisely in the middle of PSD to allow passengers to pass the train’s door conveniently. However, if all trains are changed to the new type of train, precision stop issue of train would be more important. Actually, width of the current train’s door is 1.3m and width of the widened door is 1.8m. Also, width of the PSD is 2.0m. Although the size of train’s door becomes larger than before, the size of PSD is fixed. This means that when the metro train is changed the new train which is introduced by the Korean government, the maximum tolerable stop error margin decreases from ±0.35m to ±0.1m. Therefore, more sophisticated control strategy is in need in order to meet the...
new control specification. If this precision stop issue is solved, the introduction of the new train could brings significant economic benefits to the nation with increased public transportation throughput.

1.2 Previous work

Train control has been researched for a long time and may be reaching a state of maturity in present time. Actually, there have been many previous works related to train control. For precision stop control of train, a appropriate train model is essential. However, if a train model is too complex, designing controller for a train is very difficult or may not be feasible for some advanced control technique. Also, if a train is modeled simply for designing controller, a train may not be controlled appropriately. Thus, a model which captures the essence of train dynamics yet simple enough for actual implementation is crucial for precision stop.

Among many nonlinearities which exist in a train, delayed reaction of actuator, varying degree of actuator saturation according to velocity of train, and switching of actuator are important factors to consider when a train is modeled for control purpose. Leaving any one of above factors out may result in ineffective modeling. However, to the best of authors knowledge, no previous work considers all of above matters together in terms of suitable mathematical representation fit for control purpose.

Modeling train in references [1,8,10,11,13,20,23,26,31] is not for control purpose. Although they have dynamics of a train, they are generally not satisfactory to use in controller design directly. Works in references [1, 8, 13, 20, 23] considered varying degree of actuator saturation
Chapter 1. Introduction


There are many research of control for switching system such as references [3, 4, 9, 16, 19, 24, 30]. Especially, references [3, 4, 16, 19, 24] introduced model predictive control (MPC) for switching system. References [3, 4, 19] used a mixed-logical dynamical (MLD) hybrid model for modeling a switching system, and they called switching system as hybrid system, and they proposed a mixed integer predictive control (MIPC) for MLD hybrid model control.

As mentioned above, we assume a train as an nonlinear switching system like reference [2]. However, in reference [2], actuator’s transient response were driven from the raw experimental datasheet provided by Korea Railroad Research Institute (KRRI), so a train’s actuator model is uncertain. Therefore, we can consider a train model as an uncertain linear system. References [17, 18] introduced MPC for uncertain linear system, which is called robust model predictive control.

1.3 Approach to precision stop control

As mentioned in section 1.1, the more sophisticated precision stop control strategy of train is in need in order to reduce stop position error. So we decide to use model predictive control (MPC) for precision stop control of train.

Reference [2] uses MPC for precision stop control of train. As mentioned in section 1.2, reference [2] considered actuation delay, switching of train’s actuator and nonlinearity of actuator’s saturation when model a train. So, reference [2] considered a train as a nonlinear switching system. Thus, reference [2] uses MPC for precision stop control of train only when train is near the stopping point where the dynamics of the train can be approximated with linear dynamics. For departing and cruising operations, PI controller is used instead of MPC. However, we want to use MPC for precision stop control of train in overall interval. so we need other type of MPC...
to control of nonlinear switching system.

As mentioned in section 1.2, there exist MPC for switching system such as MIPC [3, 4, 19]. However, the train’s actuator has uncertainty. Thus, we can consider a train as an uncertain linear system, although a train is a nonlinear switching system. MIPC do not consider uncertainty of plant, furthermore MIPC is heavy computationally. Therefore, we decide to apply robust MPC (RMPC), which is MPC for uncertain linear system introduced in reference [18], for precision stop control of train.

In order to design RMPC, we have to know all states of train. However, we can only obtain position and velocity of train. In other words, it is impossible to obtain the state of train except for position and velocity of train. Therefore, we select robust output feedback model predictive control (ROFMPC) [17], which is composed of RMPC and Luenberger observer, for the precision stop control of train.

### 1.4 Contribution

The main contributions of this thesis is exploring the possibility of applying robust output feedback model predictive control for precision stop control purpose of a metropolitan train in practical perspective.

### 1.5 Thesis outline

This thesis is organized as follows. Chapter 2 introduces a metropolitan train system to help comprehension of this thesis. Chapter 3 provides the model of metropolitan train. Chapter 4 briefly reviews four kinds of model predictive control, which are general model predictive control (MPC), output feedback MPC (OFMPC), robust MPC (RMPC) and robust output feedback MPC (ROFMPC) in order to help comprehension of ROFMPC. Chapter 5 describes how OFMPC and ROFMPC are designed. Chapter 6 simulates the precision stop control of train using OFMPC and ROFMPC to compare OFMPC and ROFMPC, and the precision stop control using ROFMPC in several situation. Some conclusions are drawn in Chapter 7. In Chapter 8, we compare RMPC and state-feedback control, and we simulate the state-feedback control with state estimation for precision stop control of train.
2.1 Automatic train control (ATC)

Automatic train control, or ATC, works as a safety mechanism which oversees the entire train system. ATC monitors condition of the rails and position of each train. ATC sets maximum velocity for given interval, taking consideration of various information which it gathered. The set maximum velocity is then delivery to automatic train operation.

2.2 Automatic train operation (ATO)

The most significant part of metropolitan train system is arguably automatic train operation, or ATO. ATO is in charge of providing hardware environment for train controller and communication and recording of relevant data. ATO receives position data from PSM, and deliver them to the train controllers.

2.3 Car types

There are two different kinds of train vehicle. One is 'Motor car' or 'M-car' and other is 'Trail car' or 'T-car'. The M-car has traction motor, regenerative brake and air brake. The T-car does not have traction motor and regenerative brake but has disk brakes which are generally stronger than air brakes in M-car.

The new metropolitan train model is consist of all M-cars, which makes it possible to control each car with separate control inputs. This property is exploited for control strategy. Details are explained in Section 5.1.
2.4 Train formation

Previous metropolitan trains used on Korea are consisted of lead vehicle and tail vehicle. Figure 2.1(a) shows the MT type, which is composed of M-car and T-car. An M-car plays a role of the lead vehicle and T-car plays a role of rail vehicle within a pair. Each pair receives the controller command as one unit. However, the MM type which is shown in Figure 2.1(b) is consisted entirely of M-cars. The paring is no longer necessary, because every vehicles are equipped with both the traction motor and brakes.

2.5 Precision stop marker (PSM)

There are four precision stop markers (PSM) used in Korean metropolitan train system. Each PSM has its own ID and are positioned as Figure 2.2. If there are two separate entries to one station, PSM1 is located before the rail gets separated and are shared by both rails. Operations carried out by passing each PSM is as follows. When a train pass the PSM1, the train resets remaining distance counter to $541m$. Load velocity profile depends on the current speed and stopping station. Counter continues to calculate position afterward by encoders from wheel. When the train pass the PSM2, the train proceed with control. When slowing down, only use regenerative brake. When the train pass the PSM3, the train stop using regenerative brake and
use air brake. When the train pass the PSM4, the train resets remaining distance to 3.5m. Only air brake is used. When the train arrive at the station, the train stop at a designated position.

2.6 Velocity reference generation

A metropolitan train follows premade velocity reference. There are many methods of velocity reference generation [15]. One of methods of velocity reference generation is generating velocity reference based on PSM, the maximum jerk, maximum velocity for given interval, current velocity whenever the train pass PSM. Figure 2.3 shows an example of velocity reference.
2.6. Velocity reference generation
3

Modeling

3.1 Train model

In this thesis, a train is regarded as point mass like these many studies [2,5,6,21,22,27,29]. The differential equation of a train is given by

\[ \dot{p}_v = v_v, \]
\[ m_i v_i = k_{i-1}(p_{v_{i-1}} - p_{v_i}) + k_i(p_{v_{i+1}} - p_{v_i}) + c_{i-1}(v_{v_{i-1}} - v_{v_i}) + c_i(v_{v_{i+1}} - v_{v_i}) + F_i, \]  

(3.1)

where \( m_i, p_{v_i}, v_{v_i} \) are \( i^{th} \) vehicle’s mass, position, and velocity. Parameters \( k_i, c_i \) are spring constant and damper constant of couplers of \( i^{th} \) vehicle and \( i + 1^{th} \) vehicle, and they are given by \( 5 \times 10^7 N/m \) and \( 3 \times 10^4 Ns/m \) respectively [12]. The notation \( F_i \) notates the total force exerted upon the \( i^{th} \) vehicle, and is generated by traction motor, regenerative brake, air brake, and external forces. These are described in section 3.2.

In this thesis, a train is assumed to be composed of six vehicles as shown in Figure 3.1 and each vehicle has several actuators such as traction motor, regenerative brake, and air brake. According to equation (3.1), the state-space equation of the train can be led as follows.

\[ \dot{x}_T = A_T x_T + B_T u_T, \]
\[ y_T = C_T x_T, \]  

(3.2)

where \( x_T \) is state of the train, \( u_T \) is control input of the train, \( y_T \) is output of the train, and \( x_T, \)
3.1. Train model

Figure 3.1: A train composed of six vehicles.

\( u_T, y_T, A_T, B_T, \) and \( C_T \) are given by

\[
x_T = \begin{bmatrix} p_{v1} \\ v_{v1} \\ \vdots \\ p_{v6} \\ v_{v6} \end{bmatrix}, \quad u_T = \begin{bmatrix} F_1 \\ \vdots \\ F_6 \end{bmatrix}, \quad y_T = \begin{bmatrix} p_{v1} \\ v_{v1} \\ \vdots \\ p_{v6} \\ v_{v6} \end{bmatrix}
\]

\[
A_T = \begin{bmatrix} R_{b1} & R_{c1} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} \\ R_{a2} & R_{b2} & R_{c2} & 0_{2 \times 2} & 0_{2 \times 2} \\ 0_{2 \times 2} & R_{a3} & R_{b3} & R_{c3} & 0_{2 \times 2} \\ 0_{2 \times 2} & 0_{2 \times 2} & R_{a4} & R_{b4} & R_{c4} \\ 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & R_{a5} & R_{b5} \\ 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & R_{a6} & R_{b6} \end{bmatrix},
\]

\[
R_{a1} = \begin{bmatrix} 0 & 0 \\ \frac{k_{i-1}}{m_i} & \frac{c_{i-1}}{m_i} \end{bmatrix}, \quad R_{bi} = \begin{bmatrix} 0 & 1 \\ -\frac{k_{i-1} - k_i}{m_i} & -\frac{c_{i-1} - c_i}{m_i} \end{bmatrix}, \quad R_{ci} = \begin{bmatrix} 0 & 0 \\ \frac{k_i}{m_i} & \frac{c_i}{m_i} \end{bmatrix},
\]

\[
B_T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{m_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{m_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{m_4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{m_5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{m_6} \end{bmatrix}, \quad C_T = I_{12 \times 12}.
\]
A train is influenced by many external force such as running resistance, track gradient resistance, track curve resistance, etc [11]. However, we assume that a train is only influenced by running resistance in order to simplify simulation of precision stop control of a train. The running resistance is described in section 3.5. Each vehicle’s mass is considered to have uncertainty, because vehicle’s mass may be measured inexacty.

### 3.2 Actuators of a train

Actuators of a train are composed of traction motor, regenerative brake, and air brake. Traction motor can only accelerate a train. In contrast, regenerative brake and air brake can only decelerate a train. Therefore, a train has to select an actuator according to control input of a train, so a train is a switching system.

As mentioned in section 1.2, the transient responses of the actuators were driven from the raw experimental datasheet provided by KRRI in reference [2]. By fitting the raw data with second-order transfer functions, the transient responses of traction motor, regenerative brake, and air brake were approximated. Thus, models of actuators are uncertain.

Many existing works such as references [2,5,27–29] recognize time delay of train’s actuators. The transient response of actuator is modeled as combination of pure delay and symmetric transient response in references [5,27,29]. However, references [2,28] consider the transient response of actuator has dead time and is asymmetric, because the transient response of actuator has time delay when the force is being applied, but when the force is being not applied, the transient response of actuator don’t have time delay. It is not a form of pure delay. So references [2,28] define this form of time delay as dead time. References [2,28] consider the natural frequency of actuator is different whether the force is being applied or not, i.e., the natural frequency when the force is being not applied is bigger than the natural frequency when the force is being applied. So, this transient response is defined as asymmetric transient response in references [2,28].

In this thesis, the train’s actuators are assumed to have dead time and their transient response is assumed to be asymmetric transient response like references [2,28]. So, the transfer functions of actuators, which are traction motor, regenerative brake, and air brake, are different whether the force of each actuator is being applied or not.
3.2. Actuators of a train

3.2.1 Traction motor

A traction motor is an electric motor used for propulsion of a vehicle. As mentioned above, a traction motor is assumed to be a standard second-order system and have dead time. The transient response of traction motor is asymmetric, because the transfer function of traction motor is different whether the force is being applied or not.

When the force is being applied, the transfer function of traction motor is notated as $G_{tr}^{up}(s)$, whose dead time is assumed to be 0.2 sec and natural frequency is 6.9. Thus, $G_{tr}^{up}(s)$ is given by

$$G_{tr}^{up}(s) = e^{-0.2s} \frac{6.9^2}{s^2 + 2 \times 6.9s + 6.9^2}. \quad (3.4)$$

The transfer function $G_{tr}^{up}(s)$ is simplified as pade approximation to convert exponential function to rational function. Thus, the simplified $G_{tr}^{up}(s)$ is as follows.

$$G_{tr}^{up}(s) \approx \frac{-47.61s + 476.1}{s^3 + 23.8s^2 + 185.6s + 476.1}. \quad (3.5)$$

When the force is being not applied, the transfer function of traction motor is notated as $G_{tr}^{down}(s)$, whose natural frequency is assumed to be five times bigger than natural frequency of $G_{tr}^{up}(s)$. Thus, $G_{tr}^{down}(s)$’s natural frequency is 34.5, and it is as follows.

$$G_{tr}^{down}(s) = \frac{34.5^2}{s^2 + 2 \times 34.5s + 34.5^2}. \quad (3.6)$$
Chapter 3. Modeling

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3.2.2 Regenerative brake

A regenerative brake is an energy recovery mechanism which slows a vehicle or object by converting motor’s kinetic energy. Thus, we assume the transient response of regenerative brake is same with the transient response of traction motor, so the transient response of regenerative brake is assumed to be asymmetric and have dead time.

When the force is being applied, the notation of transfer function of regenerative brake is $G_{rb}^{up}(s)$, which is same with $G_{tr}^{up}(s)$. Thus, $G_{rb}^{up}(s)$ is given by

$$G_{rb}^{up}(s) = e^{-0.2s} \frac{6.9^2}{s^2 + 2 \times 6.9s + 6.9^2} \approx \frac{-47.61s + 476.1}{s^3 + 23.8s^2 + 185.6s + 476.1}. \quad (3.7)$$

When the force is being not applied, the notation of transfer function of regenerative brake is $G_{rb}^{down}(s)$, which also is same with $G_{tr}^{down}(s)$. Thus, $G_{rb}^{down}(s)$ is given by

$$G_{rb}^{down}(s) = \frac{34.5^2}{s^2 + 2 \times 34.5s + 34.5^2}. \quad (3.8)$$

The step response of regenerative brake is shown in Figure 3.3.
3.2. Actuators of a train

3.2.3 Air brake

An air brake is a type of friction brake for vehicles in which compressed air pressing on a piston is used to apply the pressure to the brake pad needed to stop the vehicle. An air brake is assumed to be a standard second-order system and have dead time. Also the transient response of air brake is assumed to be asymmetric.

When the force is being applied, the notation of transfer function of air brake is $G_{ab}^{up}(s)$. The air brake’s natural frequency and dead time are assumed to be 2.3 and 2sec. Thus, $G_{ab}^{up}(s)$ is given by

$$G_{ab}^{up}(s) = e^{-0.2s} \frac{2.3^2}{s^2 + 2 \times 2.3s + 2.3^2}.$$  \hspace{1cm} (3.9)

The transfer function $G_{ab}^{up}(s)$ is simplified as pade approximation like $G_{tr}^{up}(s)$. Thus, $G_{ab}^{up}(s)$ can be formulated as follows.

$$G_{ab}^{up}(s) \approx \frac{-5.29s + 52.9}{s^3 + 14.6s^2 + 51.29s + 52.9}.$$  \hspace{1cm} (3.10)

When the force is being not applied, the notation of transfer function of air brake is $G_{ab}^{down}(s)$, whose natural frequency is assumed to be five times bigger than natural frequency of $G_{ab}^{up}(s)$, and does not have dead time. Thus, $G_{ab}^{down}(s)$’s natural frequency is 11.5, and $G_{ab}^{down}(s)$ is given
Chapter 3. Modeling

3.3 Nonlinearity of actuator’s saturation

Train’s actuators have saturation of the force. Degree of the actuator’s saturation varies according to train’s velocity. As shown in Figure 3.5, the traction motor, regenerative brake and air

\[ C_{ab}^{\text{down}}(s) = \frac{11.5^2}{s^2 + 2 \times 11.5s + 11.5^2}. \]  

The step response of air brake is shown in Figure 3.4.
brake decrease in maximum performance under high velocity of a train.

The traction motor’s maximum force is \(1.11 \times 10^5\, N\) when train’s velocity is smaller than \(10.74\, m/s\). The traction motor’s maximum force starts to decrease when train’s velocity is larger than \(10.74\, m/s\). The regenerative brake’s maximum force is \(1.034 \times 10^5\, N\) when train’s velocity is larger than \(2.809\, m/s\) and smaller than \(17.08\, m/s\). In the other velocity, the regenerative brake’s maximum force decreases. The air brake’s maximum force is \(4.64 \times 10^4\, N\) when train’s velocity is smaller than \(3.472\, m/s\). In the other velocity, the air brake’s maximum force is almost zero, so the air brake can decelerate the train when the train only is slow.

### 3.4 Switching of Actuator

As mentioned in previous section, a train has three kinds of actuators, which are traction motor, regenerative brake, and air brake. A train has to select actuator according to control input of it, because traction motor can only accelerate and regenerative brake, air brake can only decelerate. The brake is selected according to velocity of a train because regenerative brake can not decelerate a train when the train’s velocity is smaller than about \(2.5\, m/s\). So, algorithm of
switching of actuator is like Figure 3.6.

As shown in Figure 3.6, if the control input of a vehicle of a train is positive, traction motor is used to accelerate the vehicle. If the control input is negative, the brake is selected according to velocity of the vehicle. If velocity is bigger than 2.5m/s, regenerative brake is used to decelerate the vehicle and in the other case, air brake is used, because regenerative brake’s maximum performance is very small and air brake’s maximum performance is big relatively when the vehicle’s velocity is smaller than 2.5m/s as shown in Figure 3.5.

3.5 Running resistance

In this thesis, we consider running resistance as disturbance of a train. Running resistance is sum of wheel rolling resistance and air resistance [27]. It is usually hard to analyze mathematically, so it is founded experimentally [14]. Thus, the running resistance is approximated as second order polynomial as follows, and it is based on experimental data from the KRRI [27].

\[
F^d_1(v_{v1}) = m_1(0.002v_{v1}^2 + 0.036|v_{v1}| + 0.961) \times 10^{-3},
\]

\[
F^d_i(v_{vi}) = m_i(0.036|v_{vi}| + 0.961) \times 10^{-3}, i = 2, 3, 4, 5, 6,
\]

(3.12)

where \(F^d_i\) is running resistance which affects \(i^{th}\) vehicle, and \(v_{vi}\) is \(i^{th}\) vehicle’s velocity.
3.5. Running resistance
4 Model predictive control review

4.1 Introduction

In this thesis, ROFMPC is used for precision stop control of a train. In order to help comprehension of ROFMPC, we will briefly review model predictive control (MPC) [25], output feedback model predictive control (OFMPC), robust model predictive control (RMPC) [18], and robust output feedback model predictive control (ROFMPC) [17].

4.2 Model predictive control

The discrete-time linear time-invariant system is considered as follows.

\[
\begin{align*}
  x(k + 1) &= Ax(k) + Bu(k), \\
  y(k) &= Cx(k),
\end{align*}
\]

(4.1)

where \( x(k) \in \mathbb{R}^m \) is the current state, \( u(k) \in \mathbb{R}^n \) is the current control input, \( y(k) \in \mathbb{R}^p \) is the current output. The matrices \( (A, B, C) \in \mathbb{R}^{m \times m} \times \mathbb{R}^{n \times n} \times \mathbb{R}^{p \times m} \), the couple \((A, B)\) is assumed to be controllable, and the couple \((A, C)\) is assumed to be observable. MPC is to obtain the control input \( k_N(k) \) which minimizes the cost \( J \). Model predictive control law \( k_N(k) \) is calculated as
4.2. Model predictive control

![Block diagram of MPC](image)

Figure 4.1: Block diagram of MPC.

follows.

\[ J(x(k), u(k)) = \sum_{i=1}^{N} (x_r(k+i) - x(k+i))^T Q (x_r(k+i) - x(k+i)) + \sum_{j=0}^{N-1} u(k+j)^T R u(k+j), \]

\[ U^*(k) = \arg \min_u J(x(k), u(k)), \]

\[ U^*(k) = \begin{bmatrix} u^*(k) & u^*(k+1) & \cdots & u^*(k+N-2) & u^*(k+N-1) \end{bmatrix}^T, \]

\[ k_N(k) = u^*(k), \]

where \( x_r \) is the reference of state, \( N \) is the number of prediction horizon, and \( Q, R \) are positive definite. When MPC obtains the control input \( k_N(k) \), there is the constraint of \( x, u \). This constraint of \( x, u \) is given by

\[ x(i) \in X, \quad i = k+1, k+2, \cdots, k+N, \]

\[ u(i) \in U, \quad i = k, k+1, \cdots, k+N-1, \]

where \( X \subset \mathbb{R}^m \) is closed, and \( U \subset \mathbb{R}^n \) is compact, and each set contains the origin in its interior.

To solve this optimization problem, we have to calculate \( x(k+1), x(k+2), \cdots, x(k+N) \). \( x(k+1), x(k+2), \cdots, x(k+N) \) are obtained as follows.

\[ x(k+1) = Ax(k) + Bu(k), \]

\[ x(k+2) = Ax(k+1) + Bu(k+1) = A^2 x(k) + ABu(k) + Bu(k+1), \]

\[ \vdots \]

\[ x(k+N) = Ax(k+N-1) + Bu(k+N-1), \]

\[ = A^N x(k) + A^{N-1}Bu(k) + \cdots + ABu(k+N-2) + Bu(k+N-1), \]
where \(x(k)\) is measured from plant, and \(x(k + 1), x(k + 2), \ldots, x(k + N)\) are calculated using \(A, B, x(k), u(k), u(k + 1), \ldots, u(k + N - 1)\). Thus, we only need to get proper \(u(k), u(k + 1), \ldots, u(k + N - 1)\), which are notated by \(u^*(k), u^*(k + 1), \ldots, u^*(k + N - 1)\), to minimize cost \(J\), because \(A, B, x(k)\) is obtained from plant. Also we have to know \(A, B, x(k)\) exactly to obtain \(x(k + 1), x(k + 2), \ldots, x(k + N)\). If \(A, B, x(k)\) are uncertain, we can not obtain the proper control input \(k_N(k)\) for the plant.

### 4.3 Output feedback MPC

To obtain the control input using MPC, we have to know the current state \(x(k)\) of the plant. However, in many case, it is impossible to obtain all states. For example, in this study, plant’s state is composed of vehicle’s position, velocity, and actuator’s state. However, we can only measure vehicle’s position and velocity. So, the actuator’s state have to be estimated by state estimator such as Luenberger observer, moving horizon state estimation [7] and etc. Output feedback MPC (OFMPC) is composed of Luenberger observer and MPC [17], thus OFMPC is used when it is impossible to obtain all state of plant.

In the discrete-time linear time-invariant system which is introduced in section 4.2, this Luenberger observer is as follows.

\[
\begin{align*}
\hat{x}(k + 1) &= A\hat{x}(k) + Bu(k) + L(y(k) - \hat{y}(k)), \\
\tilde{x}(k + 1) &= (A + LC)\tilde{x}(k) = A_L\tilde{x}(k),
\end{align*}
\]

where \(\hat{x}(k)\) is the current estimated state, \(\tilde{x}\) is \(x(k) - \hat{x}(k)\), and \(L\) has to satisfy \(\rho(A_L) < 1\). If \(\tilde{x}(k)\) is zero, \(\hat{x}(k)\) represents \(x(k)\). RMPC calculates \(k_N(k)\) using \(\hat{x}(k)\) which is calculated by Luenberger observer.
4.4 Robust MPC

The uncertain discrete-time linear time-invariant system is considered as follows.

\[ x(k + 1) = Ax(k) + Bu(k) + w, \]
\[ y(k) = Cx(k) + v, \]  

(4.6)

where \( w \in \mathbb{R}^m \) is an unknown state disturbance and \( v \in \mathbb{R}^n \) is an unknown output disturbance. The disturbance \( w, v \) are assumed to be bounded, i.e.

\[ w \in \mathcal{W}, \quad v \in \mathcal{V}, \]  

(4.7)

where \( \mathcal{W}, \mathcal{V} \) is compact and contains the origin (but may not have an interior). \( x, u, y, A, B, \) and \( C \) are described in section 4.2. Also we obtain the nominal system from (4.6) as neglecting the disturbance \( w \) and \( v \).

\[ \bar{x}(k + 1) = A\bar{x}(k) + B\bar{u}(k), \]
\[ \bar{y}(k) = C\bar{x}(k), \]  

(4.8)

where \( \bar{x} \) is nominal state, \( \bar{u} \) is nominal input and \( \bar{y} \) is nominal output. The nominal system is used to design robust MPC. Thus, robust MPC’s cost \( J \) is given by

\[ J(\bar{x}(k), \bar{u}(k)) = \sum_{i=0}^{N} (x_r(k + i) - \bar{x}(k + i))^T Q(x_r(k + i) - \bar{x}(k + i)) + \sum_{j=0}^{N-1} \bar{u}(k + j)^T R\bar{u}(k + j), \]  

(4.9)
Chapter 4. Model predictive control review

where \( x_r, N, Q, R \) are described in section 4.2. Robust model predictive control law \( k^R_{N}(k) \) is calculated as follows.

\[
(x^*(k), U^*(k)) = \arg \min_{x, u} (J(x(k), u(k))),
\]

\[
\bar{U}^*(k) = \begin{bmatrix}
\bar{u}^*(k) & \bar{u}^*(k+1) & \cdots & \bar{u}^*(k+N-2) & \bar{u}^*(k+N-1)
\end{bmatrix}^T
\]

\[
k^R_{N}(k) = \bar{u}^*(k) - K(x(k) - \bar{x}^*(k)),
\]

where \( K \) satisfies \( \rho(A - BK) < 1 \). There are the constraints of \( \bar{x} \) and \( \bar{u} \), which are given by

\[
\bar{x}(i) \in \bar{X} = X \ominus S, \quad i = k, k+1, \cdots, k+N,
\]

\[
\bar{u}(i) \in \bar{U} = U \ominus KS, \quad i = k, k+1, \cdots, k+N-1,
\]

where \( \bar{X} \subset \mathbb{R}^m \) is closed, and \( \bar{U}, S \) is compact, and each set contains the origin in its interior. To solve this optimization problem, we have to calculate \( \bar{x}(k+1), \bar{x}(k+2), \cdots, \bar{x}(k+N) \). \( \bar{x}(k+1), \bar{x}(k+2), \cdots, \bar{x}(k+N) \) are obtained by (4.13).

\[
\bar{x}(k+1) = A\bar{x}(k) + B\bar{x}(k),
\]

\[
\bar{x}(k+2) = A\bar{x}(k+1) + B\bar{u}(k+1) = A^2\bar{x}(k) + AB\bar{u}(k) + B\bar{u}(k+1),
\]

\[
\vdots
\]

\[
\bar{x}(k+N) = A^N\bar{x}(k) + A^{N-1}B\bar{u}(k) + \cdots + AB\bar{u}(k+N-2) + B\bar{u}(k+N-1),
\]

where \( \bar{x}(k+1), \bar{x}(k+2), \cdots, \bar{x}(k+N) \) are decided according to \( \bar{x}(k), \bar{u}(k), \bar{u}(k+1), \cdots, \bar{u}(k+N-1) \). Thus, we only need to get proper \( \bar{x}(k), u(k), u(k+1), \cdots, u(k+N-1) \), which are notated by \( \bar{x}^*(k), u^*(k), u^*(k+1), \cdots, u^*(k+N-1) \), to minimize cost \( J \). Also we have to know \( A, B \) to obtain \( x(k+1), x(k+2), \cdots, x(k+N) \). Therefore if we know \( A, B \) exactly, we can obtain the proper control input \( k^R_{N}(k) \) for the plant.

### 4.5 Robust output feedback MPC

Robust output feedback MPC is composed of RMPC and Luenberger observer. If a plant is the uncertain discrete-time linear time-invariant system, the robust MPC is used as controller for the plant to reduce effect of plant from disturbance and control uncertain linear plant well.
Robust MPC has to know all states of plant to calculate control input for plant. However it may be impossible to know all states of plant. In such a case, the state of plant can be estimated from output of plant by Luenberger observer. Therefore, if the plant is the uncertain discrete-time linear time-invariant system and it is impossible to obtain all plant’s states, robust output feedback MPC is used to control the plant.
5

Precision stop controller design

5.1 Control strategy

As mentioned in section 3.1, the train is assumed to be composed of six vehicles, so the train model’s order is a little high, furthermore the train model is complex. Thus, it is hard to design controller using this train model. Therefore, we decide to only use a first vehicle for designing the controller. The other vehicles’s control input is calculated from the first vehicle’s control input. The first vehicle’s control input is notated by $u_{v_1}$, and $i^{th}$ vehicle’s control input is notated by $u_{v_i}$. So, $u_{v_i}$ is calculated as follows.

$$u_{v_i} = \frac{m_i}{m_1}u_{v_1},$$

(5.1)

where $m_i$ is $i^{th}$ vehicle’s mass. Figure 5.1 shows this control strategy.

As described in previous section, the first vehicle is a nonlinear switching system and has uncertainty because we can not know exact model of actuators, so the first vehicle is considered as uncertain linear system in this thesis. Also, we can not know all states of the first vehicle. It means that we can only know position and velocity of the first vehicle. Therefore we select ROFMPC for precision stop control of a train. In order to design ROFMPC, the nominal system of a vehicle, which has to be linear system simplifying the vehicle model, is needed, so nominal transfer functions of vehicle’s actuator is considered as the transfer function of vehicle’s traction motor when the force is being applied, $G_{tr}^{up}$. Therefore, the nominal transfer function of $i^{th}$ vehicle’s actuators $\bar{G}_{ac}$ is given by

$$\bar{G}_{ac,v_i}(s) = \frac{-47.61s + 476.1}{s^3 + 23.8s^2 + 185.6s + 476.1}.$$  

(5.2)
The nominal transfer function of $i^{th}$ vehicle’s actuators $\tilde{G}_{ac,v}$ is symmetric transient response contrary to real vehicle’s actuators, and has delay time which is $0.2s$. The nominal continuous time state-space equation of $i^{th}$ vehicle’s actuator is as follows.

\begin{align}
\dot{\bar{x}}_{ac,v}(t) &= \bar{A}_{ac}\bar{x}_{ac,v}(t) + \bar{B}_{ac}\bar{u}_{ac,v}(t), \\
\bar{y}_{ac,v}(t) &= \bar{C}_{ac}\bar{x}_{ac,v}(t),
\end{align}

(5.3)

where $\bar{x}_{ac,v} \in \mathbb{R}^3$ is the nominal state of $i^{th}$ vehicle’s actuator, $\bar{u}_{ac,v} \in \mathbb{R}$ is the nominal control input of $i^{th}$ vehicle’s actuator and $\bar{y}_{ac,v} \in \mathbb{R}$ is the nominal output of $i^{th}$ vehicle’s actuator. The matrices $\bar{A}_{ac}$, $\bar{B}_{ac}$ and $\bar{C}_{ac}$ are given by

\begin{align}
\bar{A}_{ac} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -476.1 & -185.61 & -23.8 \end{bmatrix}, \\
\bar{B}_{ac} &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \\
\bar{C}_{ac} &= \begin{bmatrix} 476.1 & -47.61 & 0 \end{bmatrix}.
\end{align}

(5.4)

The nominal continuous time state-space equation of $i^{th}$ vehicle is considered as follows.

\begin{align}
\dot{\bar{x}}_{v}(t) &= \bar{A}_{v}\bar{x}_{v}(t) + \bar{B}_{v}\bar{u}_{v}(t), \\
\bar{y}_{v}(t) &= \bar{C}_{v}\bar{x}_{v}(t),
\end{align}

(5.5)

where $\bar{x}_{v}$ is the nominal state of the $i^{th}$ vehicle, $\bar{u}_{v}$ is the nominal control input of $i^{th}$ vehicle and $\bar{y}_{v}$ is the nominal output of the $i^{th}$ vehicle. These are given by

\begin{align}
\bar{x}_{v} &= \begin{bmatrix} \bar{p}_{v} \\ \bar{v}_{v} \\ \bar{x}_{ac,v} \end{bmatrix} \in \mathbb{R}^5, \\
\bar{u}_{v} &= \bar{u}_{ac,v} \in \mathbb{R}, \\
\bar{y}_{v} &= \begin{bmatrix} \bar{p}_{v} \\ \bar{v}_{v} \end{bmatrix} \in \mathbb{R}^2.
\end{align}

(5.6)
where $\bar{p}_{vi}$ is the nominal position of the $i^{th}$ vehicle and $\bar{v}_{vi}$ is the nominal velocity of the $i^{th}$ vehicle. The matrices $\bar{A}_v$, $\bar{B}_v$, $\bar{C}_v$ are as follows.

$$\bar{A}_v = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.0099 & -0.001 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -476.1 & -185.61 & -23.8 \end{bmatrix}, \quad \bar{B}_v = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \bar{C}_v = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \quad (5.7)$$

To design ROFMPC, the nominal continuous time state-space equation has to be converted to the nominal discrete time state-space equation. In this thesis, the nominal discrete time state-space equation’s sampling time is assumed to be 0.1 second and the nominal discrete time state-space equation is as follows.

$$\bar{x}_{vi}(k+1) = \bar{A}_v^d \bar{x}_{vi}(k) + \bar{B}_v^d \bar{u}_{vi}(k),$$

$$\bar{y}_{vi}(k) = \bar{C}_v^d \bar{x}_{vi}(k). \quad (5.8)$$

### 5.2 Controllability and observability

Before designing ROFMPC, we have to test the controllability and observability of the first vehicle. In order to test the controllability and observability, we have to obtain the controllability
matrix $C$ and observability matrix $O$. The matrices $C$ and $O$ are as follows.

$$C = \begin{bmatrix} B_{v_1} & A_{v_1} B_{v_1} & A_{v_1}^2 B_{v_1} & A_{v_1}^3 B_{v_1} & A_{v_1}^4 B_{v_1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -0.001 & 0.0337 \\ 0 & 0 & -0.001 & 0.0337 & -0.6165 \\ 0 & 0 & 1 & -23.8 & 380.83 \\ 0 & 1 & -23.8 & 380.83 & -5122.336 \\ 1 & -23.8 & 380.83 & -5122.336 & 62556.9205 \end{bmatrix}.$$  

$$O = \begin{bmatrix} \bar{C}_{v_1} \\ \bar{C}_{v_1} A_{v_1} \\ \bar{C}_{v_1} A_{v_1}^2 \\ \bar{C}_{v_1} A_{v_1}^3 \\ \bar{C}_{v_1} A_{v_1}^4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.0099 & -0.001 & 0 \\ 0 & 0 & 0.0099 & -0.001 & 0 \\ 0 & 0 & 0.0099 & -0.001 & 0 \\ 0 & 0 & 0.4761 & 0.1856 & 0.0337 \end{bmatrix}.$$  

(5.9)

The ranks of $C$ and $O$ are full rank. It means that the first vehicle is controllable and observable.

### 5.3 Output feedback MPC design

In order to compare ROFMPC and OFMPC, we simulate the precision stop control of a train using OFMPC. In order to design MPC which is included in OFMPC, Luenberger observer has to estimate the nominal state of the first vehicle $\bar{x}_{v_1}$ from $y_{v_1}$. The discrete time state-space equation of Luenberger observer is as follows.

$$\hat{x}_{v_1}(k+1) = \bar{A}_{v_1}^d \hat{x}_{v_1}(k) + \bar{B}_{v_1}^d u_{v_1}(k) + L(y_{v_1}(k) - \bar{C}_{v_1}^d \hat{x}_{v_1}(k)),$$  

(5.10)

where $\hat{x}_{v_1}$ is the estimated nominal state of the first vehicle, $y_{v_1}$ is the output of the first vehicle. The parameter $L$ is selected by try and error method, so it is given by

$$L = \begin{bmatrix} 0.3024 & -0.0111 & -0.5367 & 3.748 & -25.8344 \\ 0.0788 & 0.5686 & -11.1086 & 82.2924 & -625.0819 \end{bmatrix}.$$  

(5.11)

When MPC is designed, $\hat{x}_{v_1}$ is used for the current state of the first vehicle. The cost $J_{OFMPC}$
of MPC is given by

\[ J_{OFMPC}(\hat{x}_{v_1}(k)) = \sum_{i=1}^{20} (x_{v_1r}(k+i) - \hat{x}_{v_1r}(k+i))^T Q (x_{v_1r}(k+i) - \hat{x}_{v_1r}(k+i)), \tag{5.12} \]

where \( x_{v_1r} \in \mathbb{R}^5 \) is the reference of vehicle’s state, \( Q \) is positive definite and they are given by

\[
\begin{bmatrix}
p_{v_1r} \\
v_{v_1r} \\
0 \\
0 \\
0
\end{bmatrix}, \quad Q =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
\tag{5.13}
\]

where \( p_{v_1r} \) is the position reference of the each vehicle and \( v_{v_1r} \) is the velocity reference of the each vehicle. As shown in equation 5.13, the cost \( J_{OFMPC} \) is composed of the sum of \( p_{v_1r} - \bar{p}_{v_1} \) and \( v_{v_1r} - \bar{v}_{v_1} \) for \( N \) steps. Also, the cost \( J_{OFMPC} \) is not related with \( \bar{x}_{ac_{v_1r}}, \bar{x}_{ac_{v_1}} \).

The constraints of \( \bar{x}_{v_1}, \bar{u}_{v_1} \) are as follows.

\[
\bar{x}_{v_1}(k) = \hat{x}_{v_1}(k),
\]

\[
sat(f_b)(v_{v_1}) \leq \bar{u}_{v_1} < sat(f_{tr})(v_{v_1}),
\tag{5.14}
\]

\[
f_b = \begin{cases} 
  f_{rb} & v_{v_1} \geq 2.5 \\
  f_{ab} & \text{others}
\end{cases}
\]

where \( sat(f_{tr})(v_{v_1}) \) means the saturation of the traction motor’s force varying according to the velocity of the first vehicle, \( sat(f_b)(v_{v_1}) \) means the saturation of the brake’s force, which is the regenerative brake or air brake, varying according to the velocity of the first vehicle, \( f_{rb} \) is the regenerative brake’s force, and \( f_{ab} \) is the air brake’s force.

Finally, the control input of the first vehicle \( u_{v_1} \) is calculated by OFMPC through equation (4.2), where \( u_{v_1} \) is \( k_N \) which is obtain as equation (4.2).

### 5.4 Robust output feedback MPC design

As mentioned in section 5.1, the nominal system of the first vehicle is considered to simplify the nonlinear switching system of the first vehicle. Also, as mentioned in section 5.3, it is impossible to obtain all states of the first vehicle. Thus, Luenberger observer is used to estimate the nominal state of the first vehicle. Thus, the Luenberger observer for ROFMPC are same with (5.10) and (5.11).
RMPC which is in ROFMPC uses the nominal state of the first vehicle \( \hat{x}_{v1} \), which is estimated by Luenberger observer, to calculate the control input of the first vehicle. The cost \( J_{ROFMPC} \) is given by

\[
J_{ROFMPC}(\hat{x}_{v1}(k)) = \sum_{i=0}^{20} (x_{v1,r}(k+i) - \hat{x}_{v1}(k+i))^T Q (x_{v1,r}(k+i) - \bar{x}_{v1}(k+i)),
\]

(5.15)

where \( x_{v1,r} \in \mathbb{R}^5 \) is the reference of the first vehicle’s state, \( Q \) is positive definite and they are given by

\[
x_{v1,r} = \begin{bmatrix}
p_{v1r} \\
v_{v1r} \\
0 \\
0 \\
0
\end{bmatrix},
Q = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

(5.16)

where \( p_{v1r} \) is the position reference of the first vehicle and \( v_{v1r} \) is the velocity reference of the first vehicle. As shown in equation (5.15), \( J_{ROFMPC} \) is composed of the sum of \( p_{v1r} - \bar{p}_{v1} \) and \( v_{v1r} - \bar{v}_{v1} \) for \( N+1 \) steps. Also, \( J_{ROFMPC} \) is not related with \( \bar{x}_{acv1}, \bar{x}_{acv1} \).

The constraints of \( \bar{x}_{v1}, \bar{u}_{v1} \) are as follows.

\[
\hat{x}_{v1}(k) - \alpha \leq \bar{x}_{v1}(k) < \hat{x}_{v1}(k) + \alpha, \quad \alpha = \begin{bmatrix}
1 & 1 & 5 & 5 & 5
\end{bmatrix}^T,
\]

\[
sat(f_{rb})(v_{v1}) \leq \bar{u}_{v1} < sat(f_{tr})(v_{v1}),
\]

\[
f_{b} = \begin{cases}
frb & \text{if } v_{v1} \geq 2.5 \\
frb & \text{others}
\end{cases}
\]

(5.17)

where \( \alpha \) is selected by try and error method. The control input of the first vehicle \( u_{v1} \) is calculated by ROFMPC through the equation (4.10). Therefore, \( u_{v1} \) is \( k_{N}^{R} \). In the equation (4.10), \( K \) is selected by try and error method and \( K \) is as follows.

\[
K = 10^5 \begin{bmatrix}
2.5758 & 4.4596 & 0.0206 & 0.0016 & 0.0001
\end{bmatrix}.
\]

(5.18)
6 Simulations

6.1 Simulation scenario

The simulation is made using MATLAB and conducts to test the performance of ROFMPC for precision stop control of a train. Specification of precision stop control of train is that the stop position error of train, which is position error when a train stops completely, is within \( \pm 0.1 \text{m} \), because the new type train’s door is widened as described in the section 1.1.

In order to test the performance of ROFMPC for precision stop control of a train, we simulate the precision stop control using ROFMPC. Secondly, we compare ROFMPC and OFMPC in performance of precision stop control of a train in order to confirm whether ROFMPC is better than OFMPC or not in performance for a nonlinear switching system.

Nextly, the simulation scenarios for testing robust performance of ROFMPC are as follows. First, ROFMPC is tested for performance under uncertain train’s mass. If the measured each vehicle’s mass is different with the real each vehicle’s mass, ROFMPC may not control the train exactly. Second, ROFMPC is tested for performance under running resistance.

Lastly, we measure the calculation time of ROFMPC in order to confirm the possibility of applying ROFMPC for precision stop control of a train.

6.2 Reference for simulation

As mentioned in 2.6, velocity reference is made in advance of starting, or it is made whenever the train pass PSM. In this simulation, we assume velocity reference to be made in advance and be fixed, also position reference is made as integrating velocity reference. Figure 6.1 shows references which is used in simulation, and these are introduced in reference [2].
6.3 Test for performance of ROFMPC

In this section, ROFMPC is tested in performance for precision stop control of a train. As shown in Figure 6.2, the tracking performance of position and velocity is appropriate. The stop position error is shown in Figure 6.3. The stop position error is 0.01227 m, and it satisfies the specification of precision stop control, because it is within ±0.1 m. Therefore, ROFMPC is appropriate for precision stop control of a train.

6.4 Comparison of ROFMPC and OFMPC

In this section, ROFMPC is compared with OFMPC in performance of precision stop control of a train. As mentioned before, the performances of ROFMPC and OFMPC are compared through simulation result of the stop position error. As shown in Figure 6.4, the stop position error of ROFMPC is smaller than the position error of OFMPC. Also, the stop position error of ROFMPC is within ±0.1 m, but the stop position error of OFMPC is not within ±0.1 m. Therefore, the performance of ROFMPC is better than the performance of OFMPC.

6.5 Test for performance of ROFMPC under uncertain train’s mass

In this section, ROFMPC is tested for the performance under uncertain train’s mass. For this test, we assume that measured value of vehicle’s mass and real value of vehicle’s mass are different as shown in the Figure 6.5. The Figure 6.6 is a graph of position error of ROFMPC in the situation which is that the measured value of vehicle’s mass and real value of each vehicle’s mass are different. The stop position error of ROFMPC is 0.02479 m, which is within ±0.1 m. This result shows that the train is controlled by ROFMPC well although the measured value and real value of the vehicle’s mass are different.

6.6 Test for performance of ROFMPC under running resistance

The train is influenced by running resistance which is described in section 3.5 while the train runs. Thus, ROFMPC is tested for performance under running resistance. In the Figure 6.7, the
stop position error of ROFMPC is 0.01152m which is within ±0.1m. This result shows that the train is controlled by ROFMPC well while the train is influenced by running resistance.

6.7 Comparison of Control input and disturbance of train

The Figure 6.8(a) shows the first vehicle’s control input and disturbance. This disturbance is the force by couplers of the first vehicle and the second vehicle. As shown in this Figure, the disturbance is almost zero, and it means that the force by couplers of the first vehicle and the second vehicle is zero, because the first vehicle’s position and velocity are same with the second vehicle’s position and velocity. Thus, through this result, the proposed control strategy do not have problem of precision stop control of a train using ROFMPC.

The Figure 6.8(b) shows the first vehicle’s control input and disturbance under uncertain train’s mass. As shown in this Figure, the disturbance is within ±4200N, which is relatively very smaller than the size of the control input. Thus, through this result, the proposed control strategy do not have problem of precision stop control of a train using ROFMPC under uncertain train’s mass.

Eventually, the proposed control strategy is appropriate for precision stop control of a train using ROFMPC.

6.8 Calculation time of ROFMPC

In order to actually apply ROFMPC for precision stop control of a train, ROFMPC’s calculation time has to be faster than the sampling time of a train. In this thesis, the sampling time of a train is assumed to be 0.1sec, so ROFMPC’s calculation time has to be smaller than 0.1sec. As shown in the Figure 6.9, ROFMPC’s calculation time is about 0.021sec, and calculation time of all steps is smaller than 0.1sec. Therefore, There is not problem about actually applying ROFMPC for precision stop control of a train.
6.8. Calculation time of ROF MPC

Figure 6.1: References of position and velocity.

(a) Position reference.

(b) Velocity reference.
Chapter 6. Simulations

(a) Position tracking of ROFMPC.

(b) Velocity tracking of ROFMPC.

Figure 6.2: The tracking performance of ROFMPC
Figure 6.3: Position error of ROF MPC.
Figure 6.4: Position error of ROFMPC and OFMPC.
Figure 6.5: Measured mass and real mass of vehicles.

Figure 6.6: Position error of ROFMPC under uncertain train’s mass.
Figure 6.7: Position error of ROF MPC under running resistance.
6.8. Calculation time of ROF MPC

(a) Comparison of the first vehicle’s control input and disturbance.

(b) Comparison of the first vehicle’s control input and disturbance under uncertain train’s mass.

Figure 6.8: Comparisons of the first vehicle’s control input and disturbance.
Figure 6.9: Calculation time of ROFMPC for the control input of the first vehicle.
6.8. Calculation time of ROF MPC
This thesis began with a modeling of the metropolitan train and reviewed MPC, OFMPC, RMPC, and ROFMPC. For modeling, consideration of dead time and asymmetric transient response of actuators were one of the most important factors. We tested to demonstrate that ROFMPC is better than OFMPC for the train which is uncertain linear system. Through simulation result, we confirmed that ROFMPC is better than OFMPC, because the stop position error of ROFMPC is smaller than the stop position error of OFMPC, and the stop position error of ROFMPC is within ±0.1m while the stop position error of OFMPC is not within ±0.1m.

We tested for the performance under uncertain train’s mass and the running resistance. In general, if the train is influenced by running resistance and uncertainty of train’s mass, the stop position error may increase. However, ROFMPC meet specification of precision stop control of train under running resistance and uncertain train’s mass. Therefore, ROFMPC is appropriate for precision stop control of a train under running resistance and uncertain train’s mass. We compared control input and disturbance of the first train in order to confirm relevance of the proposed control strategy. Through simulation result, the proposed control strategy’s relevance is confirmed, because the disturbance of the first train is relatively very small than the control input of the train although train’s mass is uncertain. In order to explore the possibility of applying ROFMPC for precision stop control of a train, we obtained ROFMPC’s calculation time. Applying ROFMPC for precision stop control of a train is possible, because ROFMPC’s calculation time is smaller than 0.1sec.
8

Further discussion

8.1 Introduction

The structure of RMPC in ROFMPC consists of a usual MPC and a state-feedback control. It is shown in equation (4.10) and (4.11). Equation (4.10) is similar a model predictive control and equation (4.11) is similar a state-feedback control. Thus, we compared RMPC and state-feedback control to find out difference of RMPC and state-feedback control.

In section 8.2, we will compare RMPC and state-feedback control for a simple second order system. We will simulate the state-feedback control with state estimation for precision stop control of train and compare the performance with that of ROFMPC in section 8.3. In section 8.4, we will conclude this discussion.

8.2 Comparison of RMPC and state-feedback control for simple model

A simple model, which is an uncertain continuous-time linear time-invariant system, is as follows.

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + B(u(t) + d(t)), \\
y(t) &=Cx(t),
\end{align*}
\]  

(8.1)

where \(x \in \mathbb{R}^2\) is the state, \(u \in \mathbb{R}\) is the control input, \(y \in \mathbb{R}\) is the output and \(d \in \mathbb{R}\) is a bounded disturbance. The matrices \(A, B, C\) are as follows.

\[
A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}.
\]  

(8.2)
In order to simplify simulation, we consider $d$ is a constant which is 10. Also, $K_{RMPC}$, $K_{SFC}$ note each K parameter of RMPC which means K parameter in equation (4.11) and K parameter of state-feedback control, and they are given by

$$K_{RMPC} = K_{SFC} = \begin{bmatrix} 4 & 4.8 \end{bmatrix}.$$  \hspace{1cm} (8.3)

Reference of state of RMPC $x_r$ and reference of output of state-feedback control $y_r$ are assumed to be constant, and are as follows.

$$x_r = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \hspace{0.5cm} y_r = 1.$$ \hspace{1cm} (8.4)

The result of RMPC and state-feedback control for simple model are shown in Figure 8.1. As shown in Figure 8.1, the step response of RMPC and state-feedback control for simple model are very similar. Especially, their transient responses are a little different, but their steady state errors are same.
Chapter 8. Further discussion

8.3 State-feedback control with state estimation for precision stop control of train

We simulate the performance of state-feedback control with state estimation for precision stop control of train. The K parameter of state-feedback control with state estimation, $K_{SFC}$ and $L_{SFC}$, which is L parameter of state-feedback control with state estimation, is considered to equate to the K parameter and L parameter of ROFMPC in equation (5.18),(5.11) in order to compare the performance of ROFMPC for precision stop control of train, and is as follows.

$$K_{SFC} = 10^3 \times \begin{bmatrix} 2.5758 & 4.4596 & 0.0206 & 0.0016 & 0.0001 \\ \end{bmatrix}$$

$$L_{SFC} = \begin{bmatrix} 0.3024 & -0.0111 & -0.5367 & 3.748 & -25.8344 \\ 0.0788 & 0.5686 & -11.1086 & 82.2924 & -625.0819 \end{bmatrix}. \quad (8.5)$$

As shown in Figure 8.2, the position tracking of state-feedback control with state estimation has delay unlike the position tracking of ROFMPC, because the control input of state-feedback control is the form of state error. However, the stop position error of state-feedback control with state estimation is 0.00448m which is within ±0.1m as shown in Figure 8.3. Thus, the state-feedback control with state estimation meets a condition for successful precision stop control of train. Figure 8.4 shows the position error of state-feedback control with state estimation for
precision stop control of train under train’s uncertain mass, and Figure 8.5 show the position error of state-feedback control with state estimation for precision stop control of train under running resistance. In Figure 8.4, 8.5, results of the stop position error are within $\pm 0.1m$, so state-feedback control with state estimation is resilient under train’s uncertain mass and running resistance. As shown in Figure 8.6, calculation time of state-feedback control with state estimation is very small, so state-feedback control with state estimation do not have the problem of calculation time.

8.4 Conclusion

In this chapter, we compared RMPC and state-feedback control for a simple second order system, as shown in the result, these two controllers are similar for the performance of simple model. We simulated the precision stop control of train with state-feedback control with state estimation. The position tracking of state-feedback control with state estimation has delay unlike the position tracking of ROFMPC, so ROFMPC is better than state-feedback control with state estimation in terms of tracking problem. However, the stop position error is 0.00448 m and is within $\pm 0.1m$, so we can say that the precision stop control of train with state-feedback control satisfies the
specification of precision stop control of a train. In terms of calculation time, state-feedback control with state estimation is better than ROF MPC.

8.5 Future work

Although we confirm that the state-feedback control is similar to RMPC, we do not simulate under various scenarios. We need to analyze why the state-feedback control with state estimation has merit for uncertain linear system. Thus, in the future, we will simulate under various scenarios, and analyze why state-feedback control is similar to RMPC and why the state-feedback control with state estimation has merit for uncertain linear system.
8.5. Future work

Figure 8.5: Position error of state-feedback control with state estimation for precision stop control of train under running resistance.

Figure 8.6: Calculation time of state-feedback control with state estimation for precision stop control of train.
A.1 Precision stop control of train with ROFMPC

Precision_stop_control_of_train_with_ROFMPC.m

```matlab
clear all;
pTs=0.1;
horizon=20;
trac_interval = 789;
reg_interval_s = 790;
reg_interval_e = 1069;
station_d=1200;
max_vel=55/3.6;
reference_slope=1/3;
end_time=150;
testr=0:pTs:end_time;
end_count=ceil(end_time/pTs);
x2r=0;
x1r=0;
five_sec=ceil(5/pTs);
testr=0:pTs:end_time;
five_second_int=0:pTs:5;
testr=(1/10*reference_slope)*testr.^2;
int_1=ceil(5/pTs); 
int_v=testr(int_1);
linear_int_dv=max_vel-2*int_v;
linear_int=0:pTs:(1/reference_slope)*linear_int_dv;
linear_int=(reference_slope)*linear_int+int_v;
int_3=-(1/10*reference_slope)*(five_second_int-5).^2+(max_vel);
int_4=-(1/10*reference_slope)*(five_second_int).^2+(max_vel);
testr=int_1+1:length(linear_int))=linear_int;
testr(int_1+length(linear_int)+1:2*int_1+length(linear_int)+1)=int_3;
ind=find(testr>max_vel);
end_first_interval=ind(1)-1;
for i=2:end_first_interval 
x2r(i)=testr(i);
x1r(i)=x1r(i-1)+(x2r(i-1)+x2r(i))*pTs/2;
```

51
end
.curve interval distance=x1r(end_first_interval);
cruise distance=station_d-2*curve_interval_distance;
end_second_interval=ceiling((cruise distance/max veloc)/pTs)+end_first_interval;
for i=end_first_interval:end_second_interval
  x2r(i)=max veloc;
  x1r(i)=x1r(i-1)+(x2r(i-1)+x2r(i))*pTs/2;
end
end_third_interval=end_second_interval+length(five_second_int);
x2r(end_second_interval+1:end_third_interval)=int_4;
for i=end_second_interval+1:end_third_interval
  x1r(i)=x1r(i-1)+(x2r(i-1)+x2r(i))*pTs/2;
end
end_fourth_interval=end_third_interval+length(linear_int);
linear_int_2=0:pTs:(1/reference slope)*linear_int_dv;
linear_int_2=-((reference slope)*linear_int_2+x2r(end_third_interval));
x2r(end_third_interval+1:end_fourth_interval)=linear_int_2;
for i=end_third_interval+1:end_fourth_interval
  x1r(i)=x1r(i-1)+(x2r(i-1)+x2r(i))*pTs/2;
end
int_5=(1/10*reference slope)*(five_second_int-5).^2;
end_fifth_interval=end_fourth_interval+length(int_5);
for i=end_fourth_interval+1:end_fifth_interval
  x1r(i)=x1r(i-1)+(x2r(i-1)+x2r(i))*pTs/2;
end
srit=find(x2r(end_second_interval:end)<2.5,1);
start_rhc_interval=end_second_interval+srit;
distance_left=station_d-x1r(start_rhc_interval);
stop_t=(3*distance_left/x2r(start_rhc_interval));
seconds=stop_t+3;
ttt=0:pTs:seconds+3;
lin_slope=x2r(start_rhc_interval)/stop_t^2;
snd ord int=0:pTs:stop t;
q_v=lin_slope*(snd ord int-stop t).^2;
q_p=lin_slope*((1/3)*snd ord int^-3-stop_t*snd ord int^-2+stop_t^2*snd ord int);
x1r = [x1r(start_rhc_interval-1) ones(1,length(q_p))*x1r(start_rhc_interval)]
  +q_p ones(1,100)*12000;
x2r = [x2r(1:start_rhc_interval-1) q_v zeros(1,100)];
t = 0:pTs:length(x1r)*pTs-pTs;
for i=2:length(t)
  x1r(i)=x1r(i-1)+(x2r(i-1)+x2r(i))*pTs/2;
end
for i = 1:length(t)-1
  x3r(i) = (x2r(i+1)-x2r(i))/pTs;
end
x3r(50) = 0.5;
x3r(308) = 0.5;
x3r(838) = -0.5;
Table=load('tables reference velocity, traction, brake_140610.mat');
Table_unt.trac_v=Table.trac_v/3.6;
Table_unt.trac_f=Table.trac_f*1000;
Table_unt.air_brake_v=Table.air_brake_v/3.6;
Table_unt.air_brake_f=Table.air_brake_f*1000;
gTable_unt.reg_brake_v=gTable.reg_brake_v/3.6;
gTable_unt.reg_brake_f=gTable.reg_brake_f*1000;
plot(gTable_unt.reg_brake_v,gTable_unt.reg_brake_f)
s = tf('s');
t_delay = 0.2;
wn_ab=2.3;
n1_ab=[0 wn_ab wn_ab^2];
d1_ab=[1 2*wn_ab wn_ab^2];
tf_ab = tf(n1_ab,d1_ab);
tf_ab = exp(-t_delay*s)*tf_ab;
tf_ab = pade(tf_ab);
[n1_ab,d1_ab] = tfdata(tf_ab,'v');
[Aab_,Bab_,Cab_,Dab_] = tf2ss(n1_ab,d1_ab);
mp1_ab=Aab_;mp2_ab=Bab_;mp3_ab=Cab_;mp4_ab=Dab_;s1_ab=ss(Aab_,Bab_,Cab_,Dab_);
s2_ab=c2d(s1_ab,pTs);
Aab_d=s2_ab.a;
Bab_d=s2_ab.b;
Cab_d=s2_ab.c;
wn_ab_dn=wn_ab*5;
tf_ab_dn = zpk([-20],[-wn_ab_dn -wn_ab_dn -20],wn_ab_dn.^2);
[n1_ab_dn,d1_ab_dn] = tfdata(tf_ab_dn,'v');
[Aab_dn,Bab_dn,Cab_dn,Dab_dn] = tf2ss(n1_ab_dn,d1_ab_dn);
mp1_ab_dn=Aab_dn;mp2_ab_dn=Bab_dn;mp3_ab_dn=Cab_dn;mp4_ab_dn=Dab_dn;
Aab_dn(1,:) = [0 1 0];
Aab_dn(2,:) = [0 0 1];
Aab_dn(3,:) = [mp1_ab(1,3) mp1_ab(1,2) mp1_ab(1,1)];
Bab_dn = [0;0;1];
Cab_dn = [mp3_ab(1,3) mp3_ab(1,2) mp3_ab(1,1)];
s1_ab_dn=ss(Aab_dn,Bab_dn,Cab_dn,Dab_dn);
s2_ab_dn=c2d(s1_ab_dn,pTs);
Aab_d_dn=s2_ab_dn.a;
Bab_d_dn=s2_ab_dn.b;
Cab_d_dn=s2_ab_dn.c;
wn_rb=2.3*3;
n1_rb=[0 wn_rb^2];
d1_rb=[1 2*wn_rb wn_rb^2];
tf_rb = tf(n1_rb,d1_rb);
tf_rb = exp(-t_delay*s)*tf_rb;
tf_rb = pade(tf_rb);
[n1_rb,d1_rb] = tfdata(tf_rb,'v');
[Arb_,Brb_,Crb_,Drb_] = tf2ss(n1_rb,d1_rb);
mp1_rb=Arb_;mp2_rb=Brb_;mp3_rb=Crb_;mp4_rb=Drb_;Arb_(1,:) = [0 1 0];
Arb_(2,:) = [0 0 1];
Arb_(3,:) = [mp1_rb(1,3) mp1_rb(1,2) mp1_rb(1,1)];
Brb_ = [0;0;1];
Crb_ = [mp3_rb(1,3) mp3_rb(1,2) mp3_rb(1,1)];
A.1. Precision stop control of train with ROFMP

```matlab
s1_rb = ss(Arb_, Brb_, Crb_, Drb_);
s2_rb = c2d(s1_rb, pTs);
Arb_d = s2_rb.a;
Brb_d = s2_rb.b;
Crb_d = s2_rb.c;

wn_rb_dn = wn_rb * 5;
tf_rb_dn = zpk([-20], [-wn_rb_dn - wn_rb_dn -20], wn_rb_dn.^2);
[n1_rb_dn, d1_rb_dn] = tfdata(tf_rb_dn,'v');
[Arb_dn, Brb_dn, Crb_dn, Drb_dn] = tf2ss(n1_rb_dn, d1_rb_dn);
mp1_rb_dn = Arb_dn;
mp2_rb_dn = Brb_dn;
mp3_rb_dn = Crb_dn;
mp4_rb_dn = Drb_dn;

Arb_dn(1,:) = [0 1 0];
Arb_dn(2,:) = [0 0 1];
Crb_dn = [mp3_rb_dn(1,3) mp3_rb_dn(1,2) mp3_rb_dn(1,1)];

s1_rb_dn = ss(Arb_dn, Brb_dn, Crb_dn, Drb_dn);
s2_rb_dn = c2d(s1_rb_dn, pTs);
Arb_d_dn = s2_rb_dn.a;
Brb_d_dn = s2_rb_dn.b;
Crb_d_dn = s2_rb_dn.c;

wn_tr = wn_rb;
n1_tr = [0 wn_tr -2];
d1_tr = [1 2*wn_tr wn_tr^2];
tf_tr = tf(n1_tr, d1_tr);
tf_tr = exp(-t_delay*s)*tf_tr;
tf_tr = pade(tf_tr);
[n1_tr, d1_tr] = tfdata(tf_tr,'v');
[Atr_, Btr_, Ctr_, Dtr_] = tf2ss(n1_tr, d1_tr);
mp1_tr = Atr_; 
mp2_tr = Btr_; 
mp3_tr = Ctr_; 
mp4_tr = Dtr_; 

wn_tr_dn = wn_tr * 5;
tf_tr_dn = zpk([-20], [-wn_tr_dn - wn_tr_dn -20], wn_tr_dn.^2);
[n1_tr_dn, d1_tr_dn] = tfdata(tf_tr_dn,'v');
[Atr_dn, Btr_dn, Ctr_dn, Dtr_dn] = tf2ss(n1_tr_dn, d1_tr_dn);
mp1_tr_dn = Atr_dn;
mp2_tr_dn = Btr_dn;
mp3_tr_dn = Ctr_dn;
mp4_tr_dn = Dtr_dn;

s1_tr = ss(Atr_, Btr_, Ctr_, Dtr_);
s2_tr = c2d(s1_tr, pTs);
Atr_(1,:) = [0 1 0];
Atr_(2,:) = [0 0 1];
Ctr_ = [mp3_tr(1,3) mp3_tr(1,2) mp3_tr(1,1)];

s1_tr_dn = ss(Atr_dn, Btr_dn, Ctr_dn, Dtr_dn);
```

\begin{verbatim}
s2_tr_dn=c2d(s1_tr_dn,pTs);
Atr_d_dn=s2_tr_dn.a;
Btr_d_dn=s2_tr_dn.b;
Ctr_d_dn=s2_tr_dn.c;
dima_one = 5;
um_car = 1;
dima = dima_one*num_car;
dimb = num_car;
n_constraints = 1;
falling_rate = -10;
m_r = [42153+6000 40463+3000 40463+5500 40463+2000 40463+4000 41990+3500];
kk=[50000000 50000000 50000000 50000000 50000000];
cc=[35000 35000 35000 35000 35000];

Ra = zeros(2,2,6);
Rb = zeros(2,2,6);
Rc = zeros(2,2,6);
Bo = zeros(2,1,6);
for i = 1:1:6
  if i==1
    Rb(:,:,i) = [0 1;-kk(i)/m_r(i) -cc(i)/m_r(i)];
    Rc(:,:,i) = [0 0;kk(i)/m_r(i) cc(i)/m_r(i)];
  elseif i==6
    Ra(:,:,i) = [0 0;kk(i-1)/m_r(i) cc(i-1)/m_r(i)];
    Rb(:,:,i) = [0 1;(-kk(i-1)-kk(i))/m_r(i) (-cc(i-1)-cc(i))/m_r(i)];
    Rc(:,:,i) = [0 0;kk(i)/m_r(i) cc(i)/m_r(i)];
  else
    Ra(:,:,i) = [0 0;kk(i-1)/m_r(i) cc(i-1)/m_r(i)];
    Rb(:,:,i) = [0 1;(-kk(i-1)-kk(i))/m_r(i) (-cc(i-1)-cc(i))/m_r(i)];
    Rc(:,:,i) = [0 0;kk(i)/m_r(i) cc(i)/m_r(i)];
  end
  Bo(:,:,i) = [0;1/m_r(i)];
end
Ao_MPC = [0 1;0 0];
Bo_MPC = [0;1/m_r(1)];
sscc = ss(Ao_MPC,Bo_MPC,eye(2),0);
ssdd = c2d(sscc,pTs);
Ado_MPC = ssdd.a;
Bdo_MPC = ssdd.b;
sscc = ss(Ao_MPC,Bo_MPC,eye(2),0);
ssdd = c2d(sscc,pTs);
Ado_MPC = ssdd.a;
Bdo_MPC = ssdd.b;
sscc = ss(Ao_MPC,Bo_MPC,eye(2),0);
ssdd = c2d(sscc,pTs);
Ado_MPC = ssdd.a;
Bdo_MPC = ssdd.b;
sscc = ss(Ao_MPC,Bo_MPC,eye(2),0);
ssdd = c2d(sscc,pTs);
Ado_MPC = ssdd.a;
Bdo_MPC = ssdd.b;
sscc = ss(Ao_MPC,Bo_MPC,eye(2),0);
ssdd = c2d(sscc,pTs);
Ado_MPC = ssdd.a;
Bdo_MPC = ssdd.b;

n_interval = length(t)-horizon-1;
Q = zeros((horizon+1)*dima+horizon*dimb,(horizon+1)*dima+horizon*dimb);
weight_tmp = [1 1 zeros(1,dima_one-2)];
weight = zeros(1,dima*(horizon+1));
for k = 1:1:horizon
  weight(1,dima*(k-1)+1:dima*k) = weight_tmp;
end
Q1:=(horizon+1)*dima+horizon+dimb) = diag(weight);
Aeq_tmp2 = [eye(dima) zeros(dima,horizon+dimb) zeros(dima,horizon*dimb)];
end
for k = 1:1:horizon
  Aeq_tmp(dima*(k-1)+1:dima*k,:) = [zeros(dima,(k-1)*dimb) Ad_MPC -eye(dima)
    zeros(dima,(horizon-k)*dimb) zeros(dima, (k-1)*dimb) Bd_MPC zeros(dima,(horizon-k)
    *dimb)];
end
Bignum = 99999;
\end{verbatim}
Aineq_tmp1 = zeros((horizon+1)*dima,(horizon+1)*dima+horizon*dimb);
for k = 1:1:horizon+1
    Aineq_tmp1((k-1)*dima+1:k*dima,:) = [zeros(dima,(k-1)*dima) -eye(dima) zeros(dima,
        (horizon+1-k)*dima) zeros(dima,horizon*dimb)];
end
Aineq_tmp2 = zeros((horizon+1)*dima,(horizon+1)*dimb+horizon*dimb);
for k = 1:1:horizon+1
    Aineq_tmp2((k-1)*dima+1:k*dima,:) = [zeros(dima,(k-1)*dima) eye(dima) zeros(dima,
        (horizon+1-k)*dima) zeros(dima,horizon*dimb)];
end
Bineq_tmp1 = zeros((horizon+1)*dima,1);
Bineq_tmp2 = zeros((horizon+1)*dima,1);
for k = 1:1:horizon+1
    Bineq_tmp1((k-1)*dima+1:k*dima,1) = [0;0;Bignum*ones(dima-2,1)];
    Bineq_tmp2((k-1)*dima+1:k*dima,1) = [Bignum;Bignum;Bignum*ones(dima-2,1)];
end
Aineq_tmp3 = zeros(horizon*dimb,(horizon+1)*dima+horizon*dimb);
Bineq_tmp3 = zeros(horizon*dimb,1);
for k = 1:1:horizon
    Aineq_tmp3((k-1)*dimb+1:k*dimb,:) = [zeros(dimb,(horizon+1)*dima) zeros(dimb,
        (k-1)*dimb) -eye(dimb) zeros(dimb,(horizon-k)*dimb)];
    Bineq_tmp3((k-1)*dimb+1:k*dimb,1) = (2*23200);
end
Aineq_tmp4 = zeros((horizon+1)*dima+(horizon+1)*dimb+horizon*dimb);
Bineq_tmp4 = zeros((horizon+1)*dimb,1);
for k = 1:1:horizon
    Aineq_tmp4((k-1)*dimb+1:k*dimb,:) = [zeros(dimb,(horizon+1)*dima) zeros(dimb,
        (k-1)*dimb) eye(dimb) zeros(dimb,(horizon-k)*dimb)];
    Bineq_tmp4((k-1)*dimb+1:k*dimb,1) = 110340;
end
Aeq = Aeq_tmp;
Beq = zeros(horizon*dima,1);
Aineq = [Aineq_tmp1;Aineq_tmp2;Aineq_tmp3;Aineq_tmp4];
Bineq = [Bineq_tmp1;Bineq_tmp2;Bineq_tmp3;Bineq_tmp4];
L_poles = [0.3 0.4 0.5 0.6 0.7];
K_poles = [0.2 0.3 0.5 0.7 0.9];
L = place(Ad_MPC’,Cd_MPC’,L_poles);
K = place(Ad_MPC,Bd_MPC,K_poles);
A1_brake = [Atr_zeros(size(Atr_,1),size(Arb_,1)) zeros(size(Atr_,1),size(Aab_,1))... zeros(size(Arb_,1),size(Atr_,1)) falling_rate*eye(size(Arb_,1))... zeros(size(Arb_,1),size(Aab_,1))... zeros(size(Aab_,1),size(Atr_,1)) zeros(size(Aab_,1),size(Arb_,1))... falling_rate*eye(size(Aab_,1))];
A2_brake = [Atr_dn zeros(size(Atr_,1),size(Arb_,1)) zeros(size(Atr_,1),size(Aab_,1))... zeros(size(Arb_,1),size(Atr_,1)) falling_rate*eye(size(Arb_,1))... zeros(size(Arb_,1),size(Aab_,1))... zeros(size(Aab_,1),size(Atr_,1)) zeros(size(Aab_,1),size(Arb_,1))... falling_rate*eye(size(Aab_,1))];
A3_brake = [falling_rate*eye(size(Atr_,1)) zeros(size(Atr_,1),size(Arb_,1))... zeros(size(Atr_,1),size(Aab_,1))... zeros(size(Aab_,1),size(Atr_,1)) Arb_zeros(size(Arb_,1),size(Aab_,1))... zeros(size(Aab_,1),size(Atr_,1)) zeros(size(Aab_,1),size(Arb_,1))... zeros(size(Aab_,1),size(Atr_,1)) zeros(size(Aab_,1),size(Arb_,1))];
falling_rate*eye(size(Aab_,1));

A4_brake = [falling_rate*eye(size(Atr_,1)) zeros(size(Atr_,1),size(Arb_,1))
zeros(size(Arb_,1),size(Atr_,1)) Arb_dn zeros(size(Arb_,1),size(Aab_,1));
zeros(size(Aab_,1),size(Atr_,1)) zeros(size(Aab_,1),size(Arb_,1))
falling_rate*eye(size(Aab_,1))];

A5_brake = [falling_rate*eye(size(Atr_,1)) zeros(size(Atr_,1),size(Arb_,1))
zeros(size(Arb_,1),size(Atr_,1)) falling_rate*eye(size(Arb_,1))
zeros(size(Arb_,1),size(Aab_,1))];

A6_brake = [falling_rate*eye(size(Atr_,1)) zeros(size(Atr_,1),size(Arb_,1))
zeros(size(Aarb_,1),size(Atr_,1)) zeros(size(Aab_,1),size(Arb_,1))
zeros(size(Aaab_,1),size(Arab_,1)) zeros(size(Aaab_,1),size(Aarb_,1))
falling_rate*eye(size(Aab_,1))];

B1_brake = [Btr_;zeros(size(Arb_,1),1);zeros(size(Aab_,1),1)];

B2_brake = [Btr_dn;zeros(size(Arb_,1),1);zeros(size(Aab_,1),1)];

B3_brake = [zeros(size(Atr_,1),1);Brb_;zeros(size(Aab_,1),1)];

B4_brake = [zeros(size(Atr_,1),1);Brb_dn;zeros(size(Aab_,1),1)];

B5_brake = [zeros(size(Atr_,1),1);zeros(size(Arb_,1),1);Bab_;]

B6_brake = [zeros(size(Atr_,1),1);zeros(size(Arb_,1),1);Bab_dn];

C1_brake = [Ctr_ zeros(size(Aarb_,1),1)];

C2_brake = [Ctr_dn zeros(size(Aarb_,1),1)];

C3_brake = [zeros(size(Atr_,1)),1);Crb_ zeros(size(Aab_,1),1));

C4_brake = [zeros(size(Atr_,1),1);Crb_dn zeros(size(Aab_,1),1)];

C5_brake = [zeros(1,size(Aarb_,1),1);zeros(1,size(Aarb_,1),1);]

C6_brake = [zeros(1,size(Aarb_,1),1);zeros(1,size(Aarb_,1),1);]

ssc1_brake = ss(A1_brake,B1_brake,C1_brake,0);

ssd1_brake = c2d(ssc1_brake,pTs);

Ad1_brake = ssd1_brake.a;

Bd1_brake = ssd1_brake.b;

Cd1_brake = ssd1_brake.c;

ssc2_brake = ss(A2_brake,B2_brake,C2_brake,0);

ssd2_brake = c2d(ssc2_brake,pTs);

Ad2_brake = ssd2_brake.a;

Bd2_brake = ssd2_brake.b;

Cd2_brake = ssd2_brake.c;

ssc3_brake = ss(A3_brake,B3_brake,C3_brake,0);

ssd3_brake = c2d(ssc3_brake,pTs);

Ad3_brake = ssd3_brake.a;

Bd3_brake = ssd3_brake.b;

Cd3_brake = ssd3_brake.c;

ssc4_brake = ss(A4_brake,B4_brake,C4_brake,0);

ssd4_brake = c2d(ssc4_brake,pTs);

Ad4_brake = ssd4_brake.a;

Bd4_brake = ssd4_brake.b;

Cd4_brake = ssd4_brake.c;

ssc5_brake = ss(A5_brake,B5_brake,C5_brake,0);

ssd5_brake = c2d(ssc5_brake,pTs);

Ad5_brake = ssd5_brake.a;

Bd5_brake = ssd5_brake.b;

Cd5_brake = ssd5_brake.c;

ssc6_brake = ss(A6_brake,B6_brake,C6_brake,0);

ssd6_brake = c2d(ssc6_brake,pTs);

Ad6_brake = ssd6_brake.a;
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\[
\text{Bd6\_brake} = \text{ssd6\_brake.b};
\]

\[
\text{Cd6\_brake} = \text{ssd6\_brake.c};
\]

\[
\text{Co\_plant} = \begin{bmatrix}
\text{eye}(2) & \text{zeros}(2,1*2) & \text{zeros}(2,4*2) & \text{zeros}(2,2*2) & \text{eye}(2) \\
\text{zeros}(2,3*2) & \ldots & \text{zeros}(2,3*2) & \text{zeros}(2,2*2) & \text{zeros}(2,4*2) & \text{zeros}(2,1*2) \\
\text{zeros}(2,5*2) & \text{eye}(2)
\end{bmatrix};
\]

\[
\text{C\_plant} = \begin{bmatrix}
\text{eye}(2) & \text{zeros}(2,\text{size}(\text{Atr}_-,1)) & \text{zeros}(2,\text{size}(\text{Arb}_-,1)) & \text{zeros}(2,\text{size}(\text{Aab}_-,1))
\end{bmatrix};
\]

\[
\text{dima\_plant} = \text{size}(\text{Ao\_MPC},1)+\text{size}(\text{Atr}_-,1)+\text{size}(\text{Arb}_-,1)+\text{size}(\text{Aab}_-,1);
\]

\[
\text{car\_num} = 6;
\]

\[
\text{theta}\_hat = \text{zeros}(\text{dima\_one},1);
\]

\[
\text{xnext}\_hat = \text{zeros}(\text{dima\_one},1);
\]

\[
\text{theta}\_bar = \text{zeros}(\text{dima\_one},1);
\]

\[
\text{theta}\_plant = \text{zeros}(\text{dima\_plant},1,6);
\]

\[
\text{xnext}\_plant = \text{zeros}(\text{dima\_plant},1,6);
\]

\[
\text{display} = \text{zeros}(\text{n\_interval},\text{dima\_plant},6);
\]

\[
\text{display}\_bar = \text{zeros}(\text{n\_interval},\text{dima\_one});
\]

\[
\text{display}\_hat = \text{zeros}(\text{n\_interval},\text{dima\_one});
\]

\[
\text{display}\_o = \text{zeros}(\text{n\_interval},12);
\]

\[
\text{display}\_f = \text{zeros}(\text{n\_interval},6);
\]

\[
\text{u} = \text{zeros}(\text{car\_num},1);
\]

\[
\text{preu} = \text{zeros}(\text{car\_num},1);
\]

\[
\text{force} = \text{zeros}(\text{car\_num},1);
\]

\[
\text{pre\_force} = \text{zeros}(\text{car\_num},1);
\]

\[
\text{force}\_tmp = \text{zeros}(\text{car\_num},1);
\]

\[
\text{f}\_\text{drag} = \text{zeros}(\text{car\_num},1);
\]

\[
\text{delta} = \text{ones}(\text{car\_num},1);
\]

\[
\text{delta}\_d = \text{ones}(\text{car\_num},1);
\]

\[
\text{theta}\_plant\_o = \text{zeros}(2*\text{car\_num},1);
\]

\[
\text{xnext}\_plant\_o = \text{zeros}(2*\text{car\_num},1);
\]

\[
\text{dev\_m} = \text{zeros}(6,1);
\]

\[
\text{m\_r\_dev} = \text{zeros}(6,1);
\]

\[
\text{for} \ k = 1:6
\]

\[
\text{dev\_m}(k,1) = \text{mod}(\text{round}(\text{rand} * 100000),31)-15)/100;
\]

\[
\text{m\_r\_dev}(k) = \text{m\_r}(k) * (1+\text{dev\_m}(k,1));
\]

\[
\text{end}
\]

\[
\text{for} \ i = 1:6
\]

\[
\text{Rb}(;,:,i) = \begin{bmatrix}
0 & 1; & -\text{kk}(i)/\text{m\_r\_dev}(i) & -\text{cc}(i)/\text{m\_r\_dev}(i)
\end{bmatrix};
\]

\[
\text{Rc}(;,:,i) = \begin{bmatrix}
0 & 0; & \text{kk}(i)/\text{m\_r\_dev}(i) & \text{cc}(i)/\text{m\_r\_dev}(i)
\end{bmatrix};
\]

\[
\text{Ra}(;,:,i) = \begin{bmatrix}
0 & 0; & \text{kk}(i-1)/\text{m\_r\_dev}(i) & \text{cc}(i-1)/\text{m\_r\_dev}(i)
\end{bmatrix};
\]

\[
\text{Rb}(;,:,i) = \begin{bmatrix}
0 & 1; & -\text{kk}(i-1)/\text{m\_r\_dev}(i) & -\text{cc}(i-1)/\text{m\_r\_dev}(i)
\end{bmatrix};
\]

\[
\text{Rc}(;,:,i) = \begin{bmatrix}
0 & 0; & \text{kk}(i)/\text{m\_r\_dev}(i) & \text{cc}(i)/\text{m\_r\_dev}(i)
\end{bmatrix};
\]

\[
\text{Bo}(;,:,i) = \begin{bmatrix}
0 & 1/\text{m\_r\_dev}(i)
\end{bmatrix};
\]

\[
\text{end}
\]

\[
\text{Ao\_plant} = \begin{bmatrix}
\text{Rb}(;,:,1) & \text{Rc}(;,:,1) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2)
\end{bmatrix};
\]

\[
\text{Rb}(;,:,2) & \text{Rc}(;,:,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2)
\end{bmatrix};
\]

\[
\text{zeros}(2,2) & \text{Rb}(;,:,3) & \text{Rc}(;,:,3) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2)
\end{bmatrix};
\]

\[
\text{zeros}(2,2) & \text{zeros}(2,2) & \text{Rb}(;,:,4) & \text{Rc}(;,:,4) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2)
\end{bmatrix};
\]

\[
\text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{Rb}(;,:,5) & \text{Rc}(;,:,5) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2)
\end{bmatrix};
\]

\[
\text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{Rb}(;,:,6) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2) & \text{zeros}(2,2)
\end{bmatrix};
\]
Bo_plant = [Bo(:,:,1) zeros(2,5);zeros(2,1) Bo(:,:,2) zeros(2,4);zeros(2,2) Bo(:,:,3);
zeros(2,3);...
zeros(2,3) Bo(:,:,4) zeros(2,2);zeros(2,4) Bo(:,:,5) zeros(2,1);zeros(2,5)
Bo(:,:,6)];
ssco = ss(Ao_plant,Bo_plant,Co_plant,0);
ssdo = c2d(ssco,pTs);
Ado_plant = ssdo.a;
Bdo_plant = ssdo.b;

xr = zeros(dima*(horizon+1),1);
for i=1:n_interval
   for k = 1:horizon+1
      xr(dima*(k-1)+1:dima*k,1) = [x1r(i+k-1);x2r(i+k-1);zeros(dima_one-2,1)];
   end
   f = -1*Q*[xr;zeros(horizon*dimb,1)];
   f_stack(i,:) = f';
   S_bar_set = [1;1;5;5;5];
   Bineq_tmp1(1:dima,1) = -theta_hat+S_bar_set;
   if Bineq_tmp1(2,1) > 0
      Bineq_tmp1(2,1) = 0;
   end
   Bineq_tmp2(1:dima,1) = theta_hat+S_bar_set;
   if Bineq_tmp2(2,1) < 0
      Bineq_tmp2(2,1) = 0;
   end
   if theta_plant(2,:,1) > 2.5
      Bineq_tmp3(1:dimb,1) = -interp1(gTable_unt.reg_brake_v,gTable_unt.reg_brake_f,
                                     theta_plant(2,:,1),'linear');
   else
      Bineq_tmp3(1:dimb,1) = (2*23200);
   end
   Bineq_tmp4(1:dimb,1) = 110340;
   Bineq = [Bineq_tmp1;Bineq_tmp2;Bineq_tmp3;Bineq_tmp4];
   theta_opt = quadprog(Q,f,Aineq,Bineq,Aeq,Beq,[],[],opts);
   u_bar = theta_opt((horizon+1)*dima+1);
   theta_bar = theta_opt(1:dima);
   u(1,1) = u_bar - K*(theta_hat-theta_bar);
   for k = 1:car_num
      if k==1
         u(k,1) = m_r(k)/m_r(1)*u(1,1);
      end
      if u(k,1) > 0
         delta(k,1) = 1;
      else
         delta(k,1) = 0;
      end
      if i~=1
         if u(k,1) > preu(k,1)
            delta_d(k,1) = 1;
         elseif u(k,1) < preu(k,1)
            delta_d(k,1) = 0;
         end
      end
      if i==1
         if delta(k,1) ==1
            (89)
xnext_plant(3:end,:,k) = Ad1_brake*theta_plant(3:end,:,k)+Bd1_brake*u(k,1);
force_tmp(k,1) = Cd1_brake*theta_plant(3:end,:,k);
if force_tmp(k,1) < pre_force(k,1)
force_tmp(k,1) = pre_force(k,1);
else
if theta_plant(2,:,k) > 2.5
xnext_plant(3:end,:,k) = Ad4_brake*theta_plant(3:end,:,k)+Bd4_brake*u(k,1);
force_tmp(k,1) = Cd4_brake*theta_plant(3:end,:,k);
xnext_plant(6:7,:,k) = xnext_plant(6:7,:,k).*Cd4_brake(4:5)';
if force_tmp(k,1) < pre_force(k,1)
force_tmp(k,1) = pre_force(k,1);
else
xnext_plant(3:end,:,k) = Ad6_brake*theta_plant(3:end,:,k)+Bd6_brake*u(k,1);
force_tmp(k,1) = Cd6_brake*theta_plant(3:end,:,k);
xnext_plant(9:10,:,k) = xnext_plant(9:10,:,k).*Cd6_brake(7:8)';
if force_tmp(k,1) < pre_force(k,1)
force_tmp(k,1) = pre_force(k,1);
else
if delta_d(k,1) == 1
if delta(k,1) ==1
theta_plant(3:4,:,k) = theta_plant(3:4,:,k)./Cd1_brake(1:2)';
xnext_plant(3:end,:,k) = Ad1_brake*theta_plant(3:end,:,k)+Bd1_brake*u(k,1);
force_tmp(k,1) = Cd1_brake*theta_plant(3:end,:,k);
xnext_plant(3:4,:,k) = xnext_plant(3:4,:,k).*Cd1_brake(1:2)';
if force_tmp(k,1) < pre_force(k,1)
force_tmp(k,1) = pre_force(k,1);
else
if theta_plant(2,:,k) >= 2.5
theta_plant(6:7,:,k) = theta_plant(6:7,:,k)./Cd4_brake(4:5)';
xnext_plant(3:end,:,k) = Ad4_brake*theta_plant(3:end,:,k)+Bd4_brake*u(k,1);
force_tmp(k,1) = Cd4_brake*theta_plant(3:end,:,k);
xnext_plant(6:7,:,k) = xnext_plant(6:7,:,k).*Cd4_brake(4:5)';
if force_tmp(k,1) < pre_force(k,1)
force_tmp(k,1) = pre_force(k,1);
else
theta_plant(9:10,:,k) = theta_plant(9:10,:,k)./Cd6_brake(7:8)';
xnext_plant(3:end,:,k) = Ad6_brake*theta_plant(3:end,:,k)+Bd6_brake*u(k,1);
force_tmp(k,1) = Cd6_brake*theta_plant(3:end,:,k);
xnext_plant(9:10,:,k) = xnext_plant(9:10,:,k).*Cd6_brake(7:8)';
if force_tmp(k,1) < pre_force(k,1)
force_tmp(k,1) = pre_force(k,1);
else
if delta(k,1) ==1
theta_plant(3:4,:,k) = theta_plant(3:4,:,k)./Cd2_brake(1:2)';
\[
xnext_{\text{plant}}(3:\text{end},,:) = A_{d2\_brake}\cdot \theta_{\text{plant}}(3:\text{end},:,k) + B_{d2\_brake}\cdot u(k,1);
\]
\[
\text{force\_tmp}(k,1) = C_{d2\_brake}\cdot \theta_{\text{plant}}(3:\text{end},:,k);
\]
\[
xnext_{\text{plant}}(3:4,:,k) = xnext_{\text{plant}}(3:4,:,k)\cdot C_{d2\_brake}(1:2)';
\]
\[
\text{if force\_tmp}(k,1) > \text{pre\_force}(k,1)
\]
\[
\text{force\_tmp}(k,1) = \text{pre\_force}(k,1);
\]
\[
\text{end}
\]
\[
\text{else}
\]
\[
\text{if } \theta_{\text{plant}}(2,:,k) >= 2.5
\]
\[
\theta_{\text{plant}}(6:7,:,k) = \theta_{\text{plant}}(6:7,:,k) / C_{d3\_brake}(4:5)';
\]
\[
xnext_{\text{plant}}(3:\text{end},,:) = A_{d3\_brake}\cdot \theta_{\text{plant}}(3:\text{end},:,k) + B_{d3\_brake}\cdot u(k,1);
\]
\[
\text{force\_tmp}(k,1) = C_{d3\_brake}\cdot \theta_{\text{plant}}(3:\text{end},:,k);
\]
\[
xnext_{\text{plant}}(6:7,:,k) = xnext_{\text{plant}}(6:7,:,k)\cdot C_{d3\_brake}(4:5)';
\]
\[
\text{if force\_tmp}(k,1) > \text{pre\_force}(k,1)
\]
\[
\text{force\_tmp}(k,1) = \text{pre\_force}(k,1);
\]
\[
\text{end}
\]
\[
\text{else}
\]
\[
\theta_{\text{plant}}(9:10,:,k) = \theta_{\text{plant}}(9:10,:,k) / C_{d5\_brake}(7:8)';
\]
\[
xnext_{\text{plant}}(3:\text{end},,:) = A_{d5\_brake}\cdot \theta_{\text{plant}}(3:\text{end},:,k) + B_{d5\_brake}\cdot u(k,1);
\]
\[
\text{force\_tmp}(k,1) = C_{d5\_brake}\cdot \theta_{\text{plant}}(3:\text{end},:,k);
\]
\[
xnext_{\text{plant}}(9:10,:,k) = xnext_{\text{plant}}(9:10,:,k)\cdot C_{d5\_brake}(7:8)';
\]
\[
\text{if force\_tmp}(k,1) > \text{pre\_force}(k,1)
\]
\[
\text{force\_tmp}(k,1) = \text{pre\_force}(k,1);
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{if } k == 1
\]
\[
\text{f\_drag}(k,1) = m_r(k) * (0.002 * \theta_{\text{plant}}(2,:,k)^2 + 0.036 * \text{abs}(\theta_{\text{plant}}(2,:,k)) + 0.961) * 10^{-3};
\]
\[
\text{else}
\]
\[
\text{f\_drag}(k,1) = m_r(k) * (0.036 * \text{abs}(\theta_{\text{plant}}(2,:,k)) + 0.961) * 10^{-3};
\]
\[
\text{end}
\]
\[
\text{force}(k,1) = \text{force\_tmp}(k,1) - \text{f\_drag}(k,1);
\]
\[
\text{if delta}(k,1) == 1
\]
\[
\text{Force\_trac\_max} = \text{interp1}(gTable\_unt.trac\_v, gTable\_unt.trac\_f, \theta_{\text{plant}}(2,:,k), 'linear');
\]
\[
\text{if force}(k,1) > \text{Force\_trac\_max}
\]
\[
\text{force}(k,1) = \text{Force\_trac\_max};
\]
\[
\text{end}
\]
\[
\text{else}
\]
\[
\text{if } \theta_{\text{plant}}(2,:,k) >= 2.5
\]
\[
\text{Force\_rb\_max} = \text{interp1}(gTable\_unt.reg\_brake\_v, gTable\_unt.reg\_brake\_f, \theta_{\text{plant}}(2,:,k), 'linear');
\]
\[
\text{if force}(k,1) < \text{Force\_rb\_max}
\]
\[
\text{force}(k,1) = \text{Force\_rb\_max};
\]
\[
\text{end}
\]
\[
\text{else}
\]
\[
\text{Force\_ab\_max} = \text{interp1}(gTable\_unt.air\_brake\_v, gTable\_unt.air\_brake\_f, \theta_{\text{plant}}(2,:,k), 'linear');
\]
\[
\text{if force}(k,1) < (2 * \text{Force\_ab\_max})
\]
\[
\text{force}(k,1) = (2 * \text{Force\_ab\_max});
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\theta_{\text{plant\_o}}(2*(k-1)+1:2*k,1) = \theta_{\text{plant}}(1:2,:,k);
\]
\[
\text{xnext\_plant\_o} = A_{d0\_plant}\cdot \theta_{\text{plant\_o}} + B_{d0\_plant}\cdot \text{force};
\]
for k = 1:1:car_num
    xnext_plant(1:2,:,k) = xnext_plant_o(2*(k-1)+1:2*k,1);
    if xnext_plant(2,k)<0
        xnext_plant(1,k) = theta_plant(1,k);
        xnext_plant(2,k) = 0;
    end
end

xnext_hat = Ad_MPC*theta_hat+Bd_MPC*u(1,1)+L’*(C_plant*theta_plant(:,:,1)-Cd_MPC
*theta_hat);
for k = 1:1:car_num
    display(i,:,k) = theta_plant(:,:,k)';
end
display_o(i,:) = theta_plant_o(:,:)';
display_f(i,:) = force';
display_bar(i,:) = theta_bar';
display_hat(i,:) = theta_hat';
pre_force = force_tmp;
preu = u;
theta_plant = xnext_plant;
theta_hat = xnext_hat;
end

figure(1);
plot(t(1:n_interval),x1r(1:n_interval));
title('Position Reference');
xlabel('time(s)');
ylabel('position(m)');
figure(2);
plot(t(1:n_interval),x2r(1:n_interval));
title('Velocity Reference');
xlabel('time(s)');
ylabel('velocity(m/s)');
figure(3)
for k = 1:1:6
    plot(t(1:n_interval),display_o(1:n_interval,2*(k-1)+1)-x1r(1:n_interval)');
    hold on;
end
str = sprintf('Robust Output Feedback MPC (Position Error)');
title(str);
xlabel('time(s)');
ylabel('position error(m)');
figure(4);
plot(t(1:n_interval),x2r(1:n_interval));
hold on;
for k = 1:1:6
    plot(t(1:n_interval),display_o(1:n_interval,2*(k-1)+2));
    hold on;
end
str = sprintf('Robust Output Feedback MPC (Velocity Tracking)');
title(str);
xlabel('time(s)');
ylabel('velocity(m/s)');
figure(5);
bar([m_r_dev m_r']);
legend('measured value', 'real value');
title('Mass of Vehicle');
ylabel('mass (Kg)');
xlabel('number of vehicle');
A.1. Precision stop control of train with ROF MPC


요약문

열차 모델링과 강인 출력 귀환 모델 예측 제어를 적용한 도시철도 정위치 정차 제어


주요이휘: 강인 출력 귀환 모델 예측 제어, 도시 철도, 정위치 정차 제어, 비선형 스위칭 시스템