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Evaluation of Disposal Techniques for Electronic Circuit Board Waste Based on Fuzzy Multi-Criteria Decision Analysis

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Abstract

The rapid increase in electronic waste demands sustainable disposal solutions for non-recyclable circuit boards. This study introduces a novel fuzzy Multi-Criteria Decision Making (MCDM) framework that integrates Stepwise Weight Assessment Ratio Analysis (SWARA) for criteria weighting and the Ranking Alternatives by Perimeter Similarity (RAPS) method for evaluating disposal options under complex Pythagorean fuzzy environment. The proposed hybrid fuzzy model uniquely addresses linguistic ambiguity, expert hesitation, and conflicting criteria common in e-waste decision environments. Key criteria such as technological feasibility, environmental impact, cost, scalability, and energy use are assessed to identify the most suitable disposal strategy. Findings highlight environmental and technological factors as the most influential, with the top-ranked disposal method demonstrating strong suitability for end-of-life circuit boards. Sensitivity and comparative analyses validate the stability and reliability of the model. Overall, the study offers a practical and methodologically original decision-support tool for advancing sustainable electronic waste management.

Keywords: Complex Pythagorean fuzzy set; E-waste; Circuit boards; Multi-criteria decision making; Ranking alternatives by perimeter similarity method

1 Introduction

Waste disposal is a major global challenge, exacerbated by urbanization, industrialization, and population growth. Electronic waste is among the most rapidly increasing and hazardous waste streams, demanding urgent attention and innovative solutions (12). Globally, electronic waste generation reaches approximately 50 million tonnes annually, driven by technological advancements, shorter product life-cycles, and increasing consumer demand for electronics (7).

Circuit boards (CBs), comprising 3-5% of electronic waste, pose a significant environmental challenge (35). These components, which provide mechanical support and electrical connections for electronic elements, contain a complex mixture of precious metals, basic metals, and toxic substances. While some CBs can be recycled to recover valuable materials like silver, gold, copper, and palladium, many are not economically feasible to recycle using traditional methods (31).

The liquidation of unprompted plates of circuits represents an especially urgent environmental challenge. These ingredients include toxic substances consisting of lead, mercury, cadmium and bromine burning retardants that represent serious risks for ecosystems and human condition when incorrectly managed. Current statistics indicate that much less than 20% of the discovered circuits are subject to proper recycling or disposal, while the rest generally ends up in landfills or processes non-formal channels in developing countries that lack sufficiently good protection and environmental protection (31). One incorrectly destroyed circuit plate can contaminate approximately eighty thousand liters of groundwater with lead on its own and create lengthy environmental damage. .

Insufficient disposal of unpromotional plates of circuits leads to serious consequences such as soil infection, pollutants, first-class air degradation and related health danger to the surrounding community. These dangerous components, when burned or destroyed in the environment outside the control, discharge poisonous materials that could strive for neurological damage, breathing problems, kidney failure and multiplication of most cancer. In addition, incorrect disposal is a good loss of final resources and creates large monetary externalities through remedial environmental costs and public health burden.

Sustainable and efficient disposal of non-reusable CBs necessitates careful consideration of economic, technical, and regulatory feasibility alongside environmental protection. The inherent difficulty in precisely quantifying these competing criteria makes this problem well-suited to fuzzy Multi-Criteria Decision Making (MCDM) approaches, which can handle imprecision and offer structured evaluations of disposal options. This research develops and applies a comprehensive fuzzy MCDM system to identify optimal, environmentally sound solutions for disposing of non-reusable CBs, mitigating their negative impacts, and addressing a critical aspect of the global e-waste issue.

Recent developments in electronic waste management increasingly rely on advanced MCDM approaches to address the complex environmental, technological, and economic challenges associated with circuit board disposal. Traditional techniques such as ELECTRE and Simple Additive Weighting offer basic evaluation frameworks but often struggle to handle uncertain, incomplete, or linguistically expressed expert data. To address these weaknesses, fuzzy MCDM methods including fuzzy SWARA for systematic criteria weighting and the Ranking Alternatives by Perimeter Similarity (RAPS) method for alternative assessment have gained prominence due to their ability to incorporate expert hesitation and ambiguity. Moreover, Complex Pythagorean Fuzzy Sets (CPFS) have recently emerged as a powerful extension of classical fuzzy models, enabling enhanced representation of membership, non-membership, and hesitation degrees in a complex plane. Despite these advancements, existing studies have not in-

egrated SWARA and RAPS within the CPFS environment for evaluating disposal techniques of non-recyclable circuit boards. This gap necessitates a more holistic, uncertainty-resilient decision-making framework to support sustainable and efficient CB disposal.

Managing non-recyclable circuit board disposal presents several key challenges that have been highlighted in prior studies. These include uncertainty in environmental impact data, inconsistent expert judgments, conflicting criteria, and incomplete or imprecise information issues that traditional decision-making methods are unable to capture effectively (Liao et al., 2023). Weight determination is another critical challenge; experts often struggle to express precise numerical preferences, making accurate prioritization of criteria difficult (Chaurasiya & Jain, 2023). Furthermore, classical fuzzy sets have limited ability to represent hesitation, leading to loss of information and reduced decision accuracy in complex environmental problems (Rani et al., 2020). Recent studies propose several implementation solutions. Fuzzy MCDM approaches, particularly the SWARA method, improve weighting reliability by allowing decision-makers to adjust preferences sequentially, making it suitable for uncertain or evolving contexts (Sahoo et al., 2024). RAPS offers a robust ranking mechanism by comparing each alternative to an ideal benchmark, improving evaluation stability even when data is incomplete. CPFS provides an advanced uncertainty modeling framework with richer membership and non-membership representations, enabling more realistic interpretation of expert hesitation. By integrating CPFS, SWARA, and RAPS, the present study addresses the main challenges identified in the literature and provides a comprehensive, uncertainty-resilient solution for sustainable circuit board disposal.

Although several studies have investigated e-waste management, existing approaches struggle to address the complexity, uncertainty, and conflicting criteria involved in selecting sustainable disposal methods for non-recyclable CBs. The majority of current MCDM research uses basic weighting-ranking models or conventional fuzzy sets, which frequently fall short in capturing linguistic ambiguity, expert hesitancy, and multi-dimensional evaluation decisions. Fuzzy extensions have been applied in some recent works, but very limited research combines CPFS with a structured hybrid MCDM model specifically designed for CBs disposal. Furthermore, no existing study integrates SWARA and RAPS under CPFS to rank disposal techniques despite its potential to offer higher precision, stronger modeling of uncertainty, and improved robustness. Thus, there is a clear lack of a holistic, uncertainty-resilient decision-support framework for evaluating environmentally and economically feasible disposal methods for non-recyclable CBs.

The motivation behind this study arises from the rapid growth of non-recyclable CBs waste, its

hazardous composition, and the severe human–environmental risks due to improper disposal. Traditional recycling processes are often costly, energy-intensive, or technologically limited, making the decision-making landscape highly complex for policymakers and industries. As global e-waste continues to surge and sustainability goals tighten, there is a pressing need for a scientifically rigorous, transparent, and robust evaluation framework. Hybrid fuzzy MCDM methods offer a promising solution because they handle uncertain expert knowledge, combine qualitative and quantitative aspects, and ensure objective criteria prioritization and reliable alternative ranking. The study is motivated by the opportunity to bring together the strengths of CPFS, SWARA, and RAPS to support accurate, sustainable, and data-consistent decision-making in CB disposal. This study makes several important contributions:

- Development of a novel hybrid fuzzy MCDM framework that integrates SWARA for criteria weighting and RAPS for alternative ranking under the CPFS environment, a combination not previously explored for e-waste disposal.
- Modeling of uncertainty, expert hesitation, and conflicting criteria with CPFS, enabling more precise representation of expert judgments compared to traditional fuzzy or crisp MCDM methods.
- Systematic evaluation of seven key criteria, reflecting a holistic and realistic view of circuit board disposal challenges.
- Practical decision-support tool for policymakers, recycling firms, and industries to adopt scientifically grounded strategies for end-of-life circuit board management.
- Identification and ranking of five major disposal alternatives, with plasma arc recycling emerging as the most sustainable option based on environmental impact, technological feasibility, and operational efficiency.
- Robustness verification through sensitivity and comparative analyses, demonstrating stability of the proposed SWARA-RAPS model under varying weight conditions.

2 Review of literature

Techniques with MCDM are critical tools used to solve complex real international problems by facilitating the evaluation of various alternatives and choosing the most suitable solution (17). Among them is the Elimination Et Choix La Realite (ELECTRE) proud of its technique, although it lacks the usually

used quantifiable characters. Another significant approach of MCDM, simple additive weighting (SAW), evaluates weighted possibilities in several criteria to distinguish alternatives of solutions. Sembiring et al., (29) declare that the aim to find the weighted preferences for all criteria. The analysis of the Step-wise Weight Assessment Ratio Analysis (SWARA) (29) also applies extensively, allowing the creators of the policy creators to optimize their decisions using criteria of evaluation in the order of significance, assigning the highest priority to the most important criteria and the lowest to the least significant (9).

The evaluation of alternatives with the help of peripheral similarity (RAPS) is a new method for decision-making (MCDM) with a multi-criteria, designed by (34). The RAPS method was introduced as a new technique by (6), supported by three major MCDM situations: the complexity involved in the evaluation of various MCDM techniques, changing the consequences provided by the same strategies and the inconsistencies listed in one of the species strategies. They argued that the proliferation of numerous MCDM strategies, limited information available to approximately newer strategies and the desire for strict validation of these strategies underlines the need for innovative approaches that include RAPS. The development of RAPS is subsequently an extensive and unusual contribution to literature on MCDM strategies (37).

Deciding on multiple criteria (MCDM) is important for choosing sustainable methods for management of non-recycled circuits boards. Scientists in various fields use MCDM strategies to perceive pure alternatives to solve problems associated with electronic waste. The primary goal of MCDM is to help creators of choosing in comparison and selecting alternatives, especially on multiple factors (34). This method is particularly effective in conditions where it fears more criteria and there is no best answer. MCDM methods are widely used in a number of industries, including engineering, health, strength, finance and training (27).

The concept of complicated fuzzy set has the mathematical framework in which Club describes the use of complicated numbers later (33). He formalized the concept of complex fuzzy elegance and organized a new notation for those orchards in which the real and imaginary components of the club function bring fuzzy statistics, which extends the idea of a complex intuitionist fuzzy set. In the complex intuitionist fuzzy units, both clubs and non-membership and lying in the unit circle are on a complicated level. As for Pythagorean Fuzzy Sets, (1) have mentioned a number of new operations and expanded the software and theoretical houses of these sets. The complex Pythagorean Fuzzy Set (CPFS) expands the Pythagorean Fuzzy set using the representation of each member and level of non-members as points in the unit disk complicated plane, allowing more nuanced modeling of uncertainty as added and ex-

plored in recent works (33). MAX et al., (18) have implemented CPF in decision-making scenarios for renewable projects and showed a realistic fee. In addition, CPFS, Carries the terms of the segment for each membership and non-Merry, which increases its functionality to symbolize the ambiguity and vagueness in decision-making, specifically, while more than one involved in mutual standards. Empirical research using Garg and Rani has shown that CPF overcome conventional strategies in solving more criteria problems (11).

The evolution in Multi Criteria Decision Making methodologies from classical approaches such as AHP, TOPSIS, and SAW toward more sophisticated hybrid and fuzzy models. This shift is driven by the increasing complexity and uncertainty inherent in sustainability problems, including waste management (14). Hybrid and fuzzy-based MCDM techniques provide enhanced capabilities to incorporate expert judgment, ambiguity, and imprecision effectively, leading to more robust and transparent decision-making frameworks emphasize that these evolving models, often incorporating nature-inspired optimization and artificial intelligence, outperform traditional methods by offering greater adaptability and accuracy in complex environments (25).

In the domain of e-waste in CBs management, recent studies demonstrate the application of advanced fuzzy extensions such as CPFS and other higher-order fuzzy models integrated within MCDM frameworks. These approaches address the multifaceted uncertainties and subjective preferences of experts in evaluating sustainable recycling and disposal strategies. Qadir et al. (22) introduced an extended TOPSIS method using complex Pythagorean fuzzy rough Dombi aggregation for partner selection in e-waste recycling, enhancing decision robustness and sensitivity to vagueness. Similarly, Rajareega (23) and Seikh & Chatterjee (28) applied complex fuzzy environments and interval-valued Fermatean fuzzy sets, respectively, to develop multi-criteria frameworks that better capture confidence levels and improve group decision-making transparency in sustainable e-waste management. These cutting-edge fuzzy MCDM models offer more nuanced and reliable rankings of management options, providing decision-makers with scientifically rigorous and interpretable tools for environmental sustainability.

The electronic waste organization is rapidly expanding and growing by approximately a 74.78 million tonnes every year, estimating that it could reach seventy 4 million tons by 2030 (18). In 2019, THRU Continennt represented the most important share of 46.4%, accompanied by America to 24.4%, Europe at 22 %, Africa to 5% and oceania 1.3 % (11). As a result, the implementation of executive electronic waste management techniques has been a critical global priority to alleviate fitness risks for every environment and the human population (30, 32).

Mairizal et., (16) has achieved a normal share in the fact that TV and computer waste includes six percentage waste as CBS plates. Waste CBs include about thirty to thirty five percent of metals, thirty-five to forty percent resistant fabric to high temperature and twenty four to thirty percent of the wax resins (13). Therefore, materials that can be poisonous are found without problems in CBS waste in addition to published plates of perimeter plates, it may be chemical epoxy resin, burning slowing and glass fiber quote. The environmental pollutant, which is caused by non-recycled CBS, is heavy metals that have brought high toxicity, bio cumulative nature and patience EV by Andooz et al., (3).

3 Preliminaries

This section defines the proposed set with new grading and fidelity functions to enrich its use & Credibility in DM procedures.

Definition 1

Let the Complex Pythagorean Fuzzy Set (CPFS) CP_F as given below (33):

$$CP_F = \{(x, Y_C(x), Z_C(x)) \mid x \in X\},$$

where $Y_C: U \rightarrow \{z_1 \in \mathbb{C} \mid |z_1| \leq 1\}$, $Z_C: U \rightarrow \{z_2 \in \mathbb{C} \mid |z_2| \leq 1\}$ such that: $\mathcal{Y}_C(x) = l_1 = d_1 + ie_1$ & $Z_C(x) = l_2 = d_2 + ie_2$ provided that: $0 \leq |l_1|^2 + |l_2|^2 \leq 1$ or alternatively: $Y_C(x) = F_C(x) \cdot e^{2\pi i \cdot W_{F_C}(x)}$, $Z_C(x) = \sqrt{T_C(x)} \cdot e^{2\pi i \cdot W_{T_C}(x)}$ which pleasing the circumstances: $0 \leq F_C^2(x) + T_C^2(x) \leq 10 \leq W_{F_C}^2(x) + W_{T_C}^2(x) \leq 1$. The hesitancy degree be as follows: $H_C(x) = R \cdot e^{2\pi i \cdot W_{R_C}(x)}$, where $R = \sqrt{1 - (|l_1| + |l_2|)}$, $W_R(x) = 1 - (W_{T_C}(x) + W_{F_C}(x))$. The pair $CP_F = (F \cdot e^{2\pi i \cdot W_T}, T \cdot e^{2\pi i \cdot W_F})$ is called a **CPyFN**.

Definition 2

Score function $score(\kappa)$ and then accuracy function $acc(\kappa)$ are:

$$score(\kappa) = \frac{1 - ((F - T)^2) - ((W_F - W_T)^2)}{2} \quad (1)$$

where $score(\kappa) \in [0, 1]$.

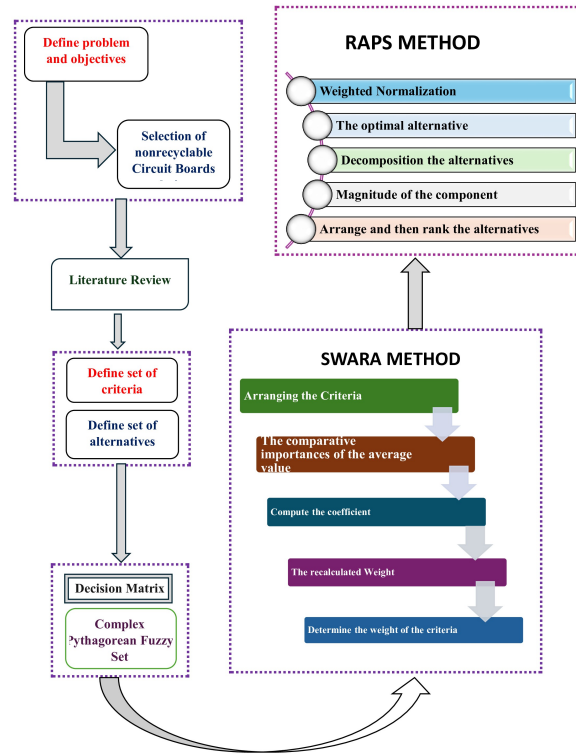


Figure 1: Proposed framework

4 Proposed methodology

This section, outlines the suggested technique for this investigation. Decision-making model for a selection of non-recyclable circuit boards, as seen in the Figure 1, combines both the RAPS and SWARA techniques in a structured system. The process starts with setting the problem and goals, followed by identification and choice of nonrecyclable circuit boards. This is then followed by an extensive literature review to guide and improve the process. After establishing the groundwork, defining the set of criteria and alternatives is the subsequent step, which is instrumental in building the decision matrix (DM) using CPFS. The DM is foundation upon which the advanced MCDM approaches, used to overcome the selection difficulty effectively.

This methodology employs SWARA technique to rigorously determine the weights of criteria. Here, in the step-by-step process, the requirements are initially ordered and their relative importance determined, which results in the calculation of coefficients and re-computation of weights to finally obtain the final criteria weights. These weights are then used in the RAPS technique, which involves adjusting the values based on their importance, breaking down the options, ranking them, figuring out their strengths, and finding the best choice. The integration of these techniques guarantees a sound, transparent, and

rigorous procedure for ranking complex alternatives and obtaining a decisive ranking for the selection of non-recyclable circuit boards.

4.1 SWARA method

The SWARA technique uses a strictly dependent sequence in which experts list and hierarchically prefer selective standards, evoke nuance and consequential comparative judgments. These qualitative knowledge is mathematically articulated as setting coefficients, allowing systematic recalibration and normalizing the weight of the criterion (2).

Step 1: The criteria are prioritized in order of their anticipated impact, from highest to lowest

Step 2: The reaction quantifies the relative importance, γ_j of each criterion j compared to the preceding criterion ($j-1$), starting from the second criterion. This ratio represents a comparative significance based on common values, providing a nuanced hierarchical weighting within the criteria set.

Step 3: The coefficient μ_j is articulated thus:

$$\mu_j = \begin{cases} 1, & j = 1 \\ \gamma_j + 1, & j > 1 \end{cases} \quad (2)$$

Step 4: The recalculating weight v_j is determined according to the following expression

$$v_j = \begin{cases} 1, & j = 1 \\ \frac{v_{j-1}}{\mu_j}, & j > 1 \end{cases} \quad (3)$$

Step 5: The relative weight of the evaluation criteria:

$$\omega_j = \frac{v_j}{\sum_{k=1}^n v_k} n \quad (4)$$

The parameter n represents the total count of criteria, whereas ω_j denotes the relative weight assigned to the j^{th} criterion, reflecting its proportional significance within the overall evaluation framework.

4.2 RAPS method

The RAPS methodology systematically normalizes, weights, evaluates, and ranks alternatives to facilitate objective decision-making across multiple criteria.

Step 1: During the first step of the RAPS methodology, input data are normalized to remove differences in units and scales, allowing for fair comparison across all criteria. This is achieved by applying specific normalization by equation (1) and (2):

$$r_{il} = \frac{x_{il}}{\max_i x_{il}}, \quad i \in [1, 2, \dots, m], l \in S_{\max} \quad (1)$$

$$r_{il} = \frac{\min_i x_{il}}{x_{il}}, \quad i \in [1, 2, \dots, m], l \in S_{\min} \quad (2)$$

the decision matrix x_{il} includes m alternatives and n criteria. Criteria are divided into two sets: S_{\max} for those to be maximized and S_{\min} for those to be minimized. This classification guides the use of specific normalization formulas to ensure all criteria are measured on a consistent, nondimensional scale for fair comparison.

Step 2: The normalized decision matrix converts all criteria values into a uniform, non dimensional scale for fair comparison are shown in the equation (3).

$$M = [m_{il}]_{k \times n} = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & \cdots & m_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ m_{k1} & m_{k2} & \cdots & m_{kn} \end{bmatrix} \quad (3)$$

Step 3: Weighted normalization equation (4) adjusts the normalized decision matrix values by multiplying each criterion's normalized value by its respective weight, reflecting the criterion's relative importance. This produces the weighted normalized matrix shown in Equation (5), which provides a dimensionless, weighted basis for evaluating and comparing alternatives more accurately according to their weighted contributions.

$$u_{ij} = w_j m_{il}, i \in [1, 2, \dots, m], l \in [1, 2, \dots, n] \quad (4)$$

$$V = [v_{il}]_{k \times n} = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{k1} & v_{k2} & \cdots & v_{kn} \end{bmatrix} \quad (5)$$

Step 4: The choice of the best alternative includes the calculation of the optimal set of opportunities

using the matrix operations equation (6). This includes multiplying the optimal components matrix with the factor matrix of each alternative to obtain an evaluation. Then the ratio of the similarity of the circuit equation (7), which compares the edge of each opportunity with an ideal alternative, allows objective evaluation. The alternative with the highest peripheral similarity is selected as the best, combining algebraic and geometric criteria for robust decisions.

$$b_j = \max v_{il} \quad 1 \leq l \leq n, i \in [1, 2, \dots, k] \quad (6)$$

$$B = \{b_1, b_2, \dots, b_l\}, l = 1, 2, \dots, n \quad (7)$$

Step 5: The optimal alternative is partitioned into two distinct subsets as expressed in Equation (8), where the subset Q represents the union of these components and k denotes their total count. The minimization criteria specify that h is the minimum number of conditions to be fulfilled. Accordingly, the optimal replacement or solution is derived using Equation (9), ensuring the best fit by satisfying the required criteria optimally within the defined subsets.

$$B = B_{\max} \cup B_{\min} \quad (8)$$

$$B = \{b_1, b_2, \dots, b_k\} \cup \{b_1, b_2, \dots, b_h\}; \quad k + h = l \quad (9)$$

Step 6: Each alternative is broken down into components following the procedure in Equations (10) and (11) for detailed analysis:

$$V_i = V_i^{\max} \cup V_i^{\min}, i \in [1, 2, \dots, k] \quad (10)$$

$$V_i = \{v_{i1}, v_{i2}, \dots, v_{ik}\} \cup \{v_{i1}, v_{i2}, \dots, v_{ih}\} \quad (11)$$

Step 7: The ideal alternative's component sizes are calculated by summing maximization and minimization criteria values, as shown in Equations (12) and (13). Similarly, the component sizes of each alternative are determined by aggregating the corresponding criterion values within these subsets, according to Equations (14) and (15). This structured partitioning allows for a systematic comparison of alternatives based on their component magnitudes before final evaluation.

$$B_k = \sqrt{b_1^2 + b_2^2 + \dots + b_k^2} \quad (12)$$

$$B_h = \sqrt{b_1^2 + b_2^2 + \dots + b_h^2} \quad (13)$$

$$V_{ik} = \sqrt{v_{i1}^2 + v_{i2}^2 + \dots + v_{ik}^2}, i \in [1, 2, \dots, k] \quad (14)$$

$$V_{ih} = \sqrt{v_{i1}^2 + v_{i2}^2 + \dots + v_{ih}^2}, i \in [1, 2, \dots, k] \quad (15)$$

Step 8: The optimal alternative's perimeter is defined as a right-angled triangle with sides Q_k and Q_h , as per Equation (16). Each alternative's perimeter is calculated similarly using Equation (17). A perimeter similarity score (PSi) is then calculated for each alternative by dividing its perimeter by the optimal perimeter, according to Equation (18). Alternatives are ranked by descending PSi, providing a geometric measure of similarity to the ideal solution.

$$O = B_k + B_h + \sqrt{B_k^2 + B_h^2} \quad (16)$$

$$O_i = V_{ik} + U_{ih} + \sqrt{V_{ik}^2 + V_{ih}^2} \quad (17)$$

$$OS_i = \frac{O_i}{O}, i \in [1, 2, \dots, m] \quad (18)$$

4.3 Theoretical and operational insights

The proposed framework is grounded in the theoretical strength of CPFS, which extend conventional fuzzy models by representing membership, non-membership, and hesitation degrees in the complex plane. This capability allows CPFS to capture multi-dimensional uncertainty, linguistic vagueness, and expert hesitation more effectively than classical or intuitionistic fuzzy sets. The operational structure of the model integrates two complementary MCDM techniques: SWARA and RAPS. SWARA provides a systematic, expert-driven approach to determine the relative importance of criteria through sequential pairwise evaluation, allowing decision-makers to adjust priorities based on contextual constraints such as environmental safety, cost, and technological feasibility. RAPS is then employed to perform alternative ranking by assessing the perimeter-based similarity between each disposal technique and the ideal solution. Together, these methods offer a transparent, mathematically consistent, and operationally flexible workflow that supports reliable evaluation of non-recyclable circuit board disposal options under uncertainty. This combined theoretical and operational foundation enables decision-makers to balance environmental, technical, and economic considerations in a structured and scientifically robust manner.

5 Case study

The fuzzy SWARA-RAPS approach is a robust tool to address the complicated multi-criteria problems of CBs waste management. Since it is able to handle uncertainty, benefit from the wisdom of experienced persons, and deliver robust rankings, it is most suitable in handling the complexity involved here. Bringing together the environmental, economic, technical, and social factors within a holistic decision framework is a very valuable improvement compared to single-objective techniques. As the field continues to grow, we envision continued methodological developments and applications across the entire e-waste management life cycle.

5.1 Criteria

Technological availability (τ_1)

Availability of suitable technology is an essential requirement for excessive waste control. Different techniques must be available, consisting of mechanical shredding and chemical recovery, to ensure that the device can apply immoderate methods for CBs that are recycled. Continuous improvement of recycling generation increases the ability to restore materials well and sustainably.

Time consumption (τ_2)

The time required to process the CBs can move drastically depending on the selected technology. However, methods such as mechanical separation may be faster, but may not bring the same heat recovery prices as extremely complicated chemical processes. Balance of speed with power is decisive for optimizing the permeability, even if it provides excessive healing costs.

Social acceptance (τ_3)

Social recognition is crucial for effective waste management. Community support and recognition programs incentivize participation in e-waste disposal regulations. Training stakeholders on the environmental and economic benefits of circuit board recycling encourages compliance and fosters a collective commitment to sustainability, strengthening awareness, responsibility, and a culture that prioritizes ecological preservation and resource conservation.

Environmental friendliness (τ_4)

The impact on the environmental effect on environmental manipulation is critical. Ecological practices include the use of tons of much less toxic substances in CBs production and implementation of environmental recycling techniques that reduce dangerous emissions and waste. The technology con-

cerning hydrometallurgical strategies is designed to obtain higher expensive metals concurrently with a reduction in environmental damage in accordance with green goals (10).

Power consumption (τ_5)

Energy efficiency is any other basic criterion. The waste inspection technology must intend to reduce energy use throughout the processing period. Techniques such as pyrolysis may also require considerable electricity input, but can also recover the strength of waste substances. The assessment of the electric trace of various strategies facilitates the selection of sustainable procedures that are in line with environmental desires.

Costs (τ_6)

The cost efficiency of the waste control is a large thing. Initial investment in technology, permanent fees for running and income from renewed material options should be considered on all. Advanced non-recycling technology, at the same time, as probably expensive in advance, can bring valuable materials that have compensated over the years. In addition, regulatory incentives for environmentally friendly procedures can decorate value efficiency. It is the very important thing for every projects as well. In the nonrecyclable CBs the cost also plays a major roll (5).

Scalability (τ_7)

The scalability refers to the capacity of the waste management strategies to solve the increasing volume of waste on the CBs without endangering efficiency. Technology, together with advanced non-recycling methods, can be extended to suit a large amount. This allows efficient processing from small batch to operational operations. It is essential that digital waste technology continues to push worldwide.

5.2 Alternatives

Landfilling (\mathfrak{I}_1)

Landfilling, a common waste disposal method involving burying waste, poses environmental risks such as soil contamination, groundwater leachate, and methane emissions during decomposition. While stringent management can mitigate these hazards, landfills are often considered a last resort due to their long-term environmental impact and potential for ecological damage.

Burning with emissions control (\mathfrak{I}_2)

The combustion includes controlled waste combustion to reduce their amount and generate energy. Modern combustion devices are prepared with excellent emissions control technologies to seize harmful pollution, such as dioxins and particles, calculated before they are listed in the surroundings. This ap-

proach can successfully reduce the use of landfills and reduce the scope of waste by up to ninety -five%, but the problems remain regarding the emissions of poisonous materials and the need for cautious monitoring to ensure adherence to environmental requirements.

The recycling of an arc in a plasma (\mathfrak{I}_3)

The recycling of the arc in the plasma uses high temperature plasma to interrupt waste substances at the molecular level. This technique is noticeably powerful for obtaining better metals from digital waste, consisting of peripheral forums, simultaneously with the conversion of organic substances into a symbol or different applicable bureaucracy. Plasma ARC technology provides a more cleaning opportunity for standard recycling methods, creating minimal emissions, and obtaining a contribution to renew valuable resources without the production of hazardous by-products.

Hydrometallurgical process (\mathfrak{I}_4)

Hydrometallurgy uses aqueous solutions to extract valuable metals from electronic waste, particularly precious metals like copper, gold and silver from peripherals by Ilyas et al., (37). This method selectively leaches target metals, followed by precipitation or solvent extraction for isolation and recovery. As it operates at lower temperatures and produces fewer emissions, hydrometallurgy is considered a more environmentally friendly alternative to pyrometallurgy.

Bioleaching (\mathfrak{I}_5)

Bioleaching uses microorganisms to extraction of metals from electronic waste through organic strategies. This approach uses the herbal abilities of positive bacteria and mushrooms to solubilize metals from ores or waste materials. Bioleaching is green and can be applied to low degrees or complicated waste currents that are difficult to use traditional techniques. However, it improves considerable industrial application, bioleaching provides a sustainable alternative for recovery from metals from circuits.

6 Results and discussion

6.1 The outcomes derived from the application of the SWARA methodology elucidate the relative importance assigned to each criterion

Step 1: The criteria are systematically ranked in descending order based on their anticipated significance.

Step 2: SWARA performance matrix is then normalized using Eq. 4 for beneficial and cost factors, respectively.

Step 3: The value of DM is computed using Eq. 5 and then normalized.

Step 4: The computed normalized matrix using mathematical software named Lingo and the final weight values are shown in Table 1 and Figure 2.

Table 1: Weights for Criteria

| Criteria | μ_j | ν_j | q_j | ω_j |
|------------------|---------|---------|----------|------------|
| \mathfrak{C}_1 | | 1 | 1 | 0.147774 |
| \mathfrak{C}_2 | 0.15 | 1.15 | 0.869565 | 0.1285 |
| \mathfrak{C}_3 | 0.3 | 1.3 | 0.884615 | 0.130724 |
| \mathfrak{C}_4 | 0.25 | 1.25 | 1.04 | 0.153685 |
| \mathfrak{C}_5 | 0.18 | 1.18 | 1.059322 | 0.156541 |
| \mathfrak{C}_6 | 0.2 | 1.2 | 0.983333 | 0.145312 |
| \mathfrak{C}_7 | 0.29 | 1.29 | 0.930233 | 0.137465 |

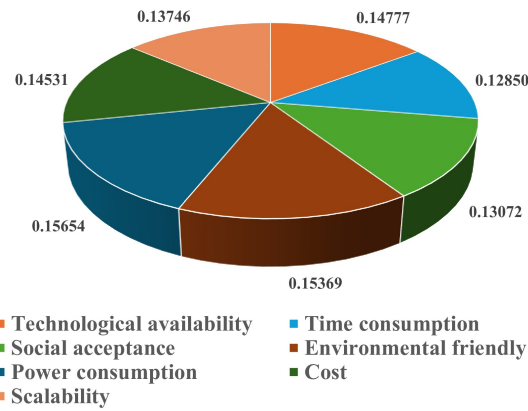


Figure 2: Ranking the Criteria

6.2 Results of the RAPS technique for ranking the ideal alternative

Step 1: A Decision Matrix (DM) in Table 2, developed following expert guidance and Definition 3, forms the basis for decision-making. This matrix evaluates six fracturing alternatives against eight conflicting criteria.

Table 2: Decision Matrix

| | \mathfrak{T}_1 | \mathfrak{T}_2 | \mathfrak{T}_3 | \mathfrak{T}_4 |
|------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|
| \mathfrak{J}_1 | $0.65e^{i0.43}, 0.27e^{i0.31}$ | $0.32e^{i0.33}, 0.78e^{i0.43}$ | $0.78e^{i0.43}, 0.45e^{i0.36}$ | $0.65e^{i0.53}, 0.32e^{i0.29}$ |
| \mathfrak{J}_2 | $0.68e^{i0.33}, 0.22e^{i0.33}$ | $0.17e^{i0.29}, 0.85e^{i0.54}$ | $0.68e^{i0.54}, 0.56e^{i0.35}$ | $0.68e^{i0.66}, 0.18e^{i0.35}$ |
| \mathfrak{J}_3 | $0.59e^{i0.66}, 0.28e^{i0.29}$ | $0.29e^{i0.26}, 0.65e^{i0.66}$ | $0.59e^{i0.66}, 0.24e^{i0.34}$ | $0.59e^{i0.74}, 0.52e^{i0.62}$ |
| \mathfrak{J}_4 | $0.77e^{i0.70}, 0.35e^{i0.26}$ | $0.35e^{i0.25}, 0.89e^{i0.54}$ | $0.87e^{i0.75}, 0.43e^{i0.26}$ | $0.74e^{i0.66}, 0.63e^{i0.62}$ |
| \mathfrak{J}_5 | $0.82e^{i0.70}, 0.45e^{i0.31}$ | $0.34e^{i0.35}, 0.75e^{i0.78}$ | $0.91e^{i0.80}, 0.34e^{i0.38}$ | $0.68e^{i0.87}, 0.61e^{i0.67}$ |

| $\bar{\tau}_5$ | $\bar{\tau}_6$ | $\bar{\tau}_7$ |
|--------------------------------|--------------------------------|--------------------------------|
| $0.22e^{i0.36}, 0.25e^{i0.53}$ | $0.32e^{i0.36}, 0.37e^{i0.53}$ | $0.71e^{i0.43}, 0.41e^{i0.36}$ |
| $0.22e^{i0.35}, 0.49e^{i0.66}$ | $0.18e^{i0.35}, 0.25e^{i0.66}$ | $0.84e^{i0.54}, 0.35e^{i0.35}$ |
| $0.28e^{i0.34}, 0.39e^{i0.74}$ | $0.29e^{i0.4}, 0.65e^{i0.74}$ | $0.69e^{i0.66}, 0.36e^{i0.34}$ |
| $0.35e^{i0.26}, 0.46e^{i0.68}$ | $0.41e^{i0.26}, 0.29e^{i0.68}$ | $0.69e^{i0.54}, 0.35e^{i0.26}$ |
| $0.41e^{i0.38}, 0.28e^{i0.87}$ | $0.37e^{i0.38}, 0.39e^{i0.87}$ | $0.89e^{i0.78}, 0.25e^{i0.38}$ |

Step 2: Subsequently, the defuzzified matrix Table 3 is normalized according to Equation 5, with the resulting values presented in Table 4.

Table 3: Defuzzified matrix

| | $\bar{\tau}_1$ | $\bar{\tau}_2$ | $\bar{\tau}_3$ | $\bar{\tau}_4$ | $\bar{\tau}_5$ | $\bar{\tau}_6$ | $\bar{\tau}_7$ |
|------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \mathfrak{J}_1 | 1 | 0.610355 | 0.933333 | 0.845078 | 0.799177 | 0.780916 | 0.714312 |
| \mathfrak{J}_2 | 0.884807 | 1 | 1 | 0.662983 | 0.79525 | 0.836598 | 0.643771 |
| \mathfrak{J}_3 | 0.911793 | 0.668778 | 0.816324 | 0.972321 | 0.91163 | 0.784138 | 0.770094 |
| \mathfrak{J}_4 | 0.74893 | 0.761012 | 0.596419 | 1 | 0.92952 | 0.907279 | 0.7473 |
| \mathfrak{J}_5 | 0.740609 | 0.734312 | 0.525224 | 0.968367 | 1 | 1 | 1 |

Table 4: Normalized decision matrix

| | $\bar{\tau}_1$ | $\bar{\tau}_2$ | $\bar{\tau}_3$ | $\bar{\tau}_4$ | $\bar{\tau}_5$ | $\bar{\tau}_6$ | $\bar{\tau}_7$ |
|------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \mathfrak{J}_1 | 1 | 0.610355 | 0.933333 | 0.845078 | 0.799177 | 0.780916 | 0.927564 |
| \mathfrak{J}_2 | 0.884807 | 1 | 1 | 0.662983 | 0.79525 | 0.836598 | 0.835963 |
| \mathfrak{J}_3 | 0.911793 | 0.668778 | 0.816324 | 0.972321 | 0.91163 | 0.784138 | 1 |
| \mathfrak{J}_4 | 0.74893 | 0.761012 | 0.596419 | 1 | 0.92952 | 0.907279 | 0.970401 |
| \mathfrak{J}_5 | 0.740609 | 0.734312 | 0.525224 | 0.968367 | 1 | 1 | 0.805458 |

Step 3: The weighted normalized matrix, resulting from the computations, is presented in Table 5.

Table 5: Weighted normalized decision matrix

| | $\bar{\tau}_1$ | $\bar{\tau}_2$ | $\bar{\tau}_3$ | $\bar{\tau}_4$ | $\bar{\tau}_5$ | $\bar{\tau}_6$ | $\bar{\tau}_7$ |
|---------------------------------|-----------------|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Weight | 0.147774 | 0.1285 | 0.130724 | 0.153685 | 0.156541 | 0.145312 | 0.137465 |
| \mathfrak{J}_1 | 0.147774 | 0.07843 | 0.122009 | 0.129876 | 0.125104 | 0.113476 | 0.098193 |
| \mathfrak{J}_2 | 0.130752 | 0.1285 | 0.130724 | 0.101891 | 0.124489 | 0.121567 | 0.088496 |
| \mathfrak{J}_3 | 0.13474 | 0.085938 | 0.106713 | 0.149432 | 0.142707 | 0.113944 | 0.105861 |
| \mathfrak{J}_4 | 0.110673 | 0.09779 | 0.077966 | 0.153685 | 0.145508 | 0.131838 | 0.102727 |
| \mathfrak{J}_5 | 0.109443 | 0.094359 | 0.068659 | 0.148824 | 0.156541 | 0.145312 | 0.137465 |
| \mathcal{Q} | 0.147774 | 0.1285 | 0.130724 | 0.153685 | 0.156541 | 0.145312 | 0.137465 |

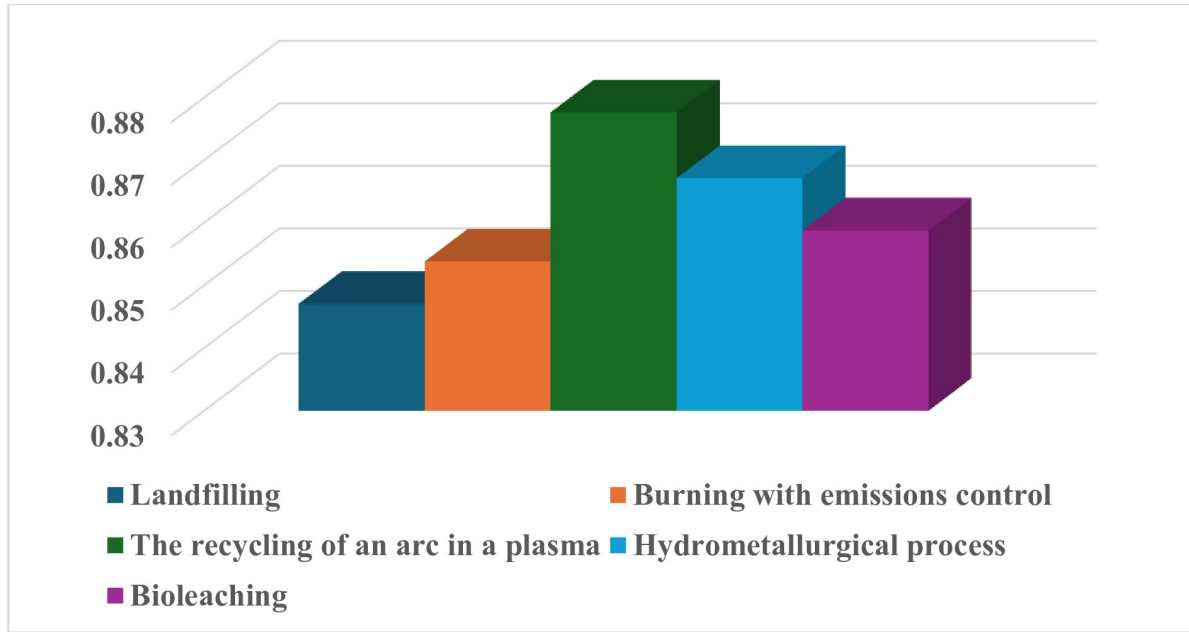


Figure 3: Ranking of the alternatives

Step 4: The optimal alternatives are subsequently determined using Equation (6), which yields the optimal alternative set as specified in Equation (7) and summarized in Table 5. (9).

Step 5: Now we move to the component magnitude for each alternative using the equation (14) and (15).

Step 6: Then now to find the perimeter by the equation (17). Finally Arrange and rank alternatives by the equation (18) are in Table 6 and Figure 3.

Table 6: Results of the SWARA-RAPS technique

| | max B_k | min B_h | o_i | OS_i | Rank |
|------------------|--------------|--------------|----------|--------|------|
| Q | 0.28538 | 0.249264 | 0.913556 | | |
| \mathfrak{J}_1 | 0.26429 | 0.186223 | 0.773822 | 0.8470 | 5 |
| \mathfrak{J}_2 | 0.240358 | 0.216306 | 0.780022 | 0.8538 | 4 |
| \mathfrak{J}_3 | 0.266024 | 0.201827 | 0.801771 | 0.8776 | 1 |
| \mathfrak{J}_4 | 0.244419 | 0.219355 | 0.792191 | 0.8672 | 2 |
| \mathfrak{J}_5 | 0.226053 | 0.233504 | 0.784555 | 0.8588 | 3 |

6.3 Sensitivity analysis

The normalization techniques are altered in the SWARA-RAPS method; we carried out the sensitivity study, in the process of normalization, the criteria units are removed, the non-beneficial criteria are zero,

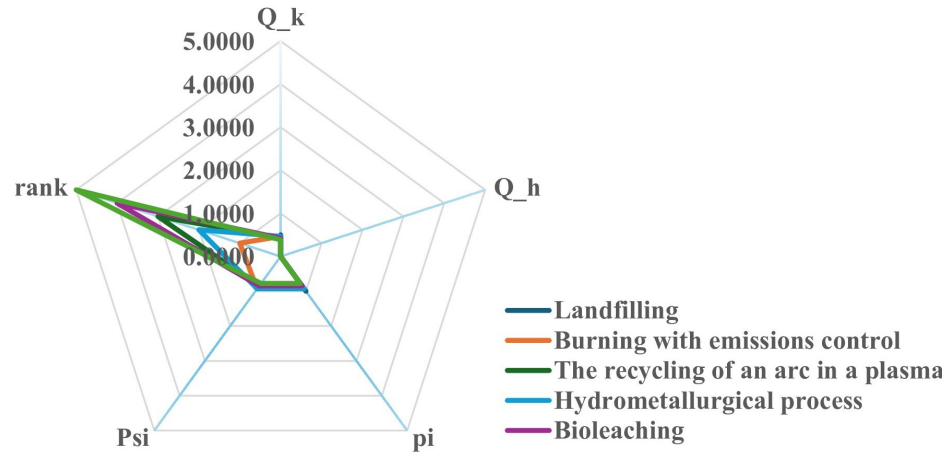


Figure 4: Radar representation of the sensitivity analysis

and then the four beneficial criteria are taken with the values of 0.25. This sensitivity analysis concluded that this problem will be changed due to the weights. So the normalized function that affected the rest of the ranking results that are shown in Table 7 and Figure 4.

Table 7: Results of the SWARA-RAPS technique

| | max Q_k | min Q_h | p_i | PS_i | Rank |
|------------------|--------------|--------------|--------|--------|------|
| Q | 0.5000 | 0.0000 | 1.0000 | | |
| \mathfrak{J}_1 | 0.4641 | 0.0000 | 0.9281 | 0.9281 | 1 |
| \mathfrak{J}_2 | 0.4273 | 0.0000 | 0.8546 | 0.8546 | 3 |
| \mathfrak{J}_3 | 0.4639 | 0.0000 | 0.9278 | 0.9278 | 2 |
| \mathfrak{J}_4 | 0.4227 | 0.0000 | 0.8453 | 0.8453 | 4 |
| \mathfrak{J}_5 | 0.3882 | 0.0000 | 0.7763 | 0.7763 | 5 |

6.4 Comparative analysis

We compare the findings from various MCDMs with those from the SWARA-RAPS method. The CPFS set is extended to the MCDM models like RAMS, WASPAS, and MABAC. All these methods are maintained with the same initial values from the DM and the weights. In the Table 8, summarize this analysis. They unequivocally show that supercritical \mathfrak{J}_3 is the preferable alternative in every single one of the existing models. The results show that the integrated MCDM framework had more flexible outcomes than the individual approaches.

Table 8: Comparison of RAMS, WASPAS, MABAC, and SWARA-RAPS Methods

| Alternatives | RAMS | WASPAS | MABAC | SWARA-RAPS |
|------------------|------------------|------------------|------------------|------------------|
| \mathfrak{J}_1 | 0.8532 | 0.9266 | 0.0262 | 0.8470 |
| \mathfrak{J}_2 | 0.8533 | 0.9177 | 0.0230 | 0.8538 |
| \mathfrak{J}_3 | 0.8812 | 0.9339 | 0.1245 | 0.8776 |
| \mathfrak{J}_4 | 0.8667 | 0.9125 | 0.0380 | 0.8672 |
| \mathfrak{J}_5 | 0.8577 | 0.9072 | 0.0385 | 0.8588 |
| Optimal Ranking | \mathfrak{J}_3 | \mathfrak{J}_3 | \mathfrak{J}_3 | \mathfrak{J}_3 |

6.5 Discussion

The increasing amount of non-recyclable CBs in e-waste necessitates innovative decomposition processes. Conventional methods often fail to simultaneously meet environmental, economic, and technological demands. This study addresses this gap by presenting a fuzzy-based decision-making approach for rigorously assessing alternative options, with a focus on multidimensional decomposability. By combining CPFS with the SWARA-RAPS method, this research offers a robust and adaptable framework for handling the vagueness and complexity inherent in e-waste recycling decisions.

The CPFS component enhances the model's ability to manage vague information and conflicting expert opinions. The SWARA approach reduces human subjectivity by objectively determining the criteria with relative weights. Integrated with the RAPS technique, hybrid approach maintains ranking accuracy by inhibiting information loss, thereby promoting the reliability of decision results. This is particularly effective when considering multiple conflicting objectives, making it suitable for assessing the CB's recycling process with fuzzy data.

Applying seven criteria environmental impact, energy demands, costs, social acceptance, scalability, time efficiency, and technological readiness to five decomposition concepts revealed that plasma arc recycling (\mathfrak{J}_3) scored highest (0.8776). Its advantages include high metal recovery, minimal hazardous emissions, no toxic by-products, and alignment with circular economy principles, making it ideal for large-scale industrial applications. Hydrometallurgy (\mathfrak{J}_4) ranked second (0.8672), efficient in precious metal recovery but limited by environmental and waste concerns due to chemical reagent use. Bioleaching (\mathfrak{J}_5), while environmentally friendly, ranked third (0.8588) due to its slow processing speed, rendering it more suitable for small-scale programs.

Emission-controlled burning and landfilling were the least favorable e-waste management methods due to the former's high energy use and low social acceptability and the latter's unsustainable disposal of valuable resources. This aligns with the global shift towards efficient, low-impact recycling and the

increasing rejection of landfilling.

Plasma arc recycling (\mathfrak{I}_3) is the preferred method for non-recyclable CBs, with a weight value of 0.8776. This is due to its superior metal recovery compared to landfilling, controlled burning, hydrometallurgical processes, and bioleaching. It offers a cleaner recycling approach with minimal emissions, yielding new renewable resources without hazardous byproducts.

Sensitivity analysis confirmed the model's stability across different criterion weights, demonstrating the robustness and reliability of the SWARA-RAPS technique. While plasma arc recycling performs best globally, its feasibility depends on specific regulations and opportunities. For instance, hydrometallurgical processes are suitable where advanced chemical waste facilities are available, and bioleaching is viable when processing time and environmental concerns are less critical. Consequently, despite plasma arc recycling's general superiority, local techno-socio-economic factors significantly influence the practical selection of a recycling method.

7 Managerial implications

The proposed fuzzy MCDM framework offers valuable insights for both academic and practical applications in electronic waste management. From an academic perspective, the integration of SWARA for criteria weighting with the RAPS method under a fuzzy environment addresses challenges such as linguistic ambiguity, expert hesitation, and conflicting criteria. This methodological contribution provides a robust reference framework for future research on complex decision-making problems, particularly in sustainability-focused domains where uncertainty and multidimensional evaluation are critical.

From a practical standpoint, the framework equips policymakers, environmental agencies, and municipal authorities with a systematic approach to identify the most sustainable and feasible disposal strategies for non-recyclable circuit boards. By prioritizing disposal methods based on technological feasibility, environmental impact, energy efficiency, cost, and scalability, the model facilitates more informed and transparent decision-making, minimizing subjective biases and improving overall effectiveness in e-waste management initiatives.

In the industrial context, the framework supports manufacturers, recycling firms, and electronics companies in selecting environmentally friendly and economically viable disposal techniques. It aids in strategic planning for resource allocation, compliance with environmental regulations, and adoption of sustainable practices, thereby enhancing operational efficiency while reducing the ecological footprint.

of electronic waste. Overall, the proposed model bridges the gap between academic research and real-world application, offering actionable guidance for sustainable decision-making in the management of end-of-life circuit boards. Additionally, advancements in computational and optimization theory such as semigroup-based optimal control models (4) and computationally efficient scientific algorithms for complex systems (20) provide methodological perspectives that can inspire deeper analytical enrichment of such decision-making frameworks.

8 Conclusion

E-waste, particularly non-recyclable circuit boards (CBs), poses a significant environmental challenge. Utilizing these CBs as a resource can substantially reduce time, cost, and power consumption while promoting technological availability, social acceptance, environmental friendliness, and scalability. However, selecting the optimal method for processing non-recyclable CBs remains complex.

This study introduces a fuzzy-based framework that integrates expert judgment to address this challenge. By employing CPFS, the framework effectively manages the uncertainty and complexity inherent in decision-making processes. Seven key criteria technological availability, time consumption, social acceptance, environmental friendliness, power consumption, cost, and scalability play a crucial role in the ranking process. The SWARA-RAPS method facilitates a robust ranking of alternatives, identifying plasma arc recycling as the most suitable method for non-recyclable CBs. Sensitivity and comparative analyses validate the robustness and efficiency of this approach. Its limited sensitivity underscores the importance of weighting criteria in ranking alternatives. This research benefits researchers, industry professionals, and policymakers in making informed decisions regarding sustainable methods for non-recyclable CBs.

Future work will explore integrating other fuzzy sets with the proposed framework and extending its application to multidisciplinary decision environments. Although the model is robust, it is not a life cycle assessment and therefore does not capture long-term and dynamic environmental impacts. Future enhancements may also include incorporating real-time data to reflect evolving conditions more accurately, allowing the framework to become a more adaptive and reliable tool for sustainability evaluation. Moreover, upcoming studies may complement the fuzzy MCDM structure with advanced optimization-oriented computational methods. Recent developments such as modified iterative PINN algorithms for strongly coupled convection-diffusion-reaction systems (21) offer powerful mathematical and numerical

tools that could be integrated into future sustainability assessment models to improve modeling depth and dynamic predictive capabilities. Integrating these advanced approaches could enhance accuracy, enrich system insights, and support more informed decision-making across diverse sectors while fostering collaborative, socially acceptable, and environmentally responsible strategies for long-term resilience.

Author Contributions

Babu Ijaz: Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Samayan Narayanamoorthy:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Methodology, Investigation, Formal analysis, Conceptualization. **Michael Sandra:** Writing – review & editing, Visualization, Validation, Software. **Naif Almakayeel:** Writing – review & editing, Visualization, Validation, Software. **Hasan Dincer:** Writing – review & editing, Validation, Software, Resources. **Serhat Yuksel:** Writing – review & editing, Visualization, Software, Resources, Funding acquisition. **Daekook Kang:** Writing – review & editing, Visualization, Validation, Software, Resources, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability

The study utilizes secondary data, which is available with the corresponding authors and can be made available upon reasonable request.

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